Abstract

The electrical systems with several nonlinear elements have more than one DC point. Newton method is very slow and determining of all dc points is not guaranteed. We can perform PWL approximation of nonlinear characteristics. The PWL characteristics can be separated into linear regions. One DC operation point can be included in this linear region. The paper deals with mathematical modeling and DC fixed points determining.

1 Introduction

A lot of mathematical models were developed for modeling piecewise-linear function. We can recall same fundamental types as explicit ones (Chua, Güzelis, Kahlert, Huertas) or implicitly given (van Bokhoven I, van Bokhoven II). These models have different properties and we can find further informations in [1]. Let’s imagine that there are many nonlinear elements in the circuit. It’s individual AV characteristics are approximated by means of piecewise linear segments described by one of the mentioned manner. The process of searching the dc solutions of large network is complex problem with extreme demands on computation technology. For example, the circuitry implementation with three nonlinear resistive elements need the solution space with three dimensions. Coordinate axis correspond to independent variables (voltage or current) of nonlinear elements. Each of these characteristics must have at least two breakpoints, which leads to solution space with 27 and more regions and the wanted solution must lie within it. Our task is to find subspace with desired solution effectively and determine it’s strict value.

Effectiveness of used method is in discard as many regions as possible which doesn’t satisfy necessary conditions for existence of solution in the early stage of the computation. We are focused mainly on method so called sign testing.

2 Creating of system equations

Let’s suppose the system with n nonlinearities modeled by using PWL. Then it is possible to decompose system into several parts, namely linear N - gate part and individual PWL elements connected to linear block. It is demonstrated in the figure 1 and deeply discussed in [2].

![Fig. 1. Execution non-linear element to from a linear n-port N.](image_url)
Mentioned system can be described by hybrid matrix equation

\[
\begin{bmatrix}
\mathbf{i}_a \\
\mathbf{v}_b
\end{bmatrix} = \begin{bmatrix}
\mathbf{H}_{aa} & \mathbf{H}_{ab} \\
\mathbf{H}_{ba} & \mathbf{H}_{bb}
\end{bmatrix} \begin{bmatrix}
\mathbf{v}_a \\
\mathbf{i}_b
\end{bmatrix} + \begin{bmatrix}
\mathbf{s}_a \\
\mathbf{s}_b
\end{bmatrix}.
\] (1)

where

\[
\begin{align*}
\mathbf{v}_a &= [\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_i]^T, \\
\mathbf{v}_b &= [\vec{v}_{i+1}, \vec{v}_{i+2}, \ldots, \vec{v}_n]^T \\
\mathbf{i}_a &= [\vec{i}_1, \vec{i}_2, \ldots, \vec{i}_i]^T, \\
\mathbf{i}_b &= [\vec{i}_{i+1}, \vec{i}_{i+2}, \ldots, \vec{i}_n]^T
\end{align*}
\] (2)

and \([\mathbf{s}_a, \mathbf{s}_b]^T\) denote the source vector due to the independent sources.

Let’s suppose nonlinear characteristics modeled by Chua’s canonical form, in detail

\[
f(x) = a + B \cdot x + \sum_{i=1}^{p} c_i |x - \beta_i|,
\] (3)

where \(c\) is constant vector and \(\beta\) is vector of breakpoints of PWL characteristics. Substituting the term (1) into the general system description given by (3) the resulting equation can be rewritten as

\[
f(x) = g(x) + H \cdot x - s = 0,
\] (4)

where \(x = (x_1, x_2, \ldots, x_n)^T \in \mathbb{R}^n\) is vector of voltages or currents in PWL resistors, \(s = (s_1, s_2, \ldots, s_n)^T \in \mathbb{R}^n\) is a source vector, \(H \in \mathbb{R}^{n \times n}\) denotes the hybrid matrix and \(g(x) = [g_1(x_1), g_2(x_2), \ldots, g_n(x_n)]^T\) is a continuous PWL function.

### 3 Algorithm for dc point location

The entire solution space is divided in individual coordinates in accordance with breakpoints of PWL characteristics. It leads to concretization of subspace where only linear shapes of characteristics are allowed. We can separate given function \(f(x)\) as follows

\[
f(x) = f_1(x_1) + f_2(x_2) + \ldots + f_n(x_n),
\] (5)

where \(f_i(x_i)\) denotes a PWL function in variable \(x_i\). This function preserves linear (constant) dependences on other variables. This method is based on researching and eliminating solutionless linear regions in step-by-step manner. It follows directly from the previous that success of this process is guaranteed in finite number of steps (finite computation time).

For better comparison we will show it on simple \(n = 3\) example. It gives rise to 3D space with proper solution. The procedure of such searching is illustrated on the figure 2.
In first method’s step we separate the initial solution space in the points corresponding to breakpoints of the function $f_i$. It is shown on the figure 2a. This is the way to split PWL function on linear sectors into direction of the axis $x_1$. As it is pointed above by definition (4) there are maximally linear dependences on others variables. For individual boxes we are in need to perform the sign test and determine the existence of solution. Particular principle of the sign rule is discussed in the next chapter. The regions satisfying the rule are conserved for further computations. The next step is division of remaining regions in direction of the axis $x_2$, see the figure 2b. Similar test is done, but we must prove simultaneously if the previous restriction is fulfilled in the same box. Analogous procedure can be applied to higher order solution spaces.

4 The sign rule principle

The solution of linear equations can be considered as

$$f(x) = 0.$$ \hspace{1cm} (6)

It is sufficient to determine the maximal and minimal value of the function (6) on the interval of linear region and establish the intersection or contact with zero. Let’s suppose function $f_i(x_1)$ in the form

$$f_i(x_1) = a + \sum_{i=1}^{n} q_i \cdot x_i + \sum_{j=1}^{p} c_j |x_i - \beta_j|,$$ \hspace{1cm} (7)

where $q_i = (q_1, q_2, \ldots, q_n) \in R^n$ is the vector containing only constants associated with linear terms. While searching for the extremes of a given $f_i$ on some linear regions, the boundary points are also defined. Absolute value term can be recasted into more compact expression for studying interval. Then the equation (7) becomes

$$f_i(x_1) = a' + \sum_{i=1}^{n} q_i' \cdot x_i,$$ \hspace{1cm} (8)

where $q_i'$ corresponds to $q_i$ but with changed constant value on the position appropriate with index of the function in (8) as absolute value terms are rearranged.
For individual independent variables and linear segments we obtain the set of boundary points well suited for examination the minimal and maximal values of whole function. If we rewrite equation (8) for three PWL elements, as it is expected in the case of our example, we get immediately

\[ f_i(x_i) = a + q_1 x_1 + q_2 x_2 + q_3 x_3. \]  

(9)

Based on the signs associated with coefficients \( q_i \) we are able to specify which boundary point leads to minimal value of (9) and which one corresponds to maximal value of (9). The table below

<table>
<thead>
<tr>
<th>Condition</th>
<th>Minimum value of ( f_i(x_i) )</th>
<th>Maximum value of ( f_i(x_i) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positive ( q_i )</td>
<td>( \text{min} )</td>
<td>( \text{Max} )</td>
</tr>
<tr>
<td>Negative ( q_i )</td>
<td>( \text{max} )</td>
<td>( \text{Min} )</td>
</tr>
</tbody>
</table>

If the following formulate is satisfied then the dc solution lie inside this element.

\[ \max\{f_i(x_i)\} \geq 0, \quad \min\{f_i(x_i)\} \leq 0. \]  

(10)

5 Conclusion

In the case when transistor in the circuit is replaced by an Ebers-Moll large signal equivalent model, it is possible to use presented method for the purpose of finding all dc solutions. It is evident that nonlinear AV characteristics of used diodes are here replaced by PWL characteristics with one breakpoint. It leads to low number of regions, in which we are forced to search the dc stationary point.

The process described above was implemented in MATLAB. Many mathematical computations were done for various distinct networks. The effectivity of this approach seems to be relatively good.

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References

