EXPERIMENTAL OBSERVATIONS OF STRANGE ATTRACTORS
IN THE MODIFIED LORENZ SYSTEM

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Abstract
This paper greatly extends the recently published experimental results obtained by means of the chaotic oscillator, which represents a modified Lorenz dynamical system. Very good final agreement between numerical analysis and practical measurements is proved via a gallery of the oscilloscope screenshots.

1. Mathematical model
First of all, let consider the following set of differential equations, which represents the so-called modified Lorenz dynamical system

\[
\begin{align*}
\frac{dx}{dt} &= -Ax - y^2 - z^2 + AC, \\
\frac{dy}{dt} &= x(y - Bz) + D, \\
\frac{dz}{dt} &= -z + x(By + z),
\end{align*}
\]

where \( A, B, C \) and \( D \) are considered as an adjustable parameters.

Note that such a dynamical system has smooth and strongly nonlinear vector field, making a deep analysis (analytical solution) almost impossible. Classical approach to the determination of the local behavior suggests to perform a linearization of the flow in the neighborhood of its equilibria and then establish a corresponding eigenvalues. In [1], this procedure as well as fixed point migration and eigenvalue changes is presented. We should recall that there can be up to five fixed points. In the region of some interest (from the viewpoint of chaos generation), there are just three unstable equilibria of saddle-focus type. By looking to the state equations, it is clear that modified Lorenz system is not an example of simple chaotic flow having six or less terms and a single nonlinearity. Some of these canonical systems were published in [3] and realized as an electronic circuit in [4].

2. Electronic circuit
A suitable tool for modeling the qualitative behavior of the modified Lorenz dynamical system was proposed in the publication [2]. But, from the practical viewpoint, only the most common chaotic attractor is covered by this article. This one will be omitted. We adopt the following listing of the circuit parameters: \( C_1 = C_2 = C_3 = 100\text{nF}, \) fixed and variable linear resistors with the nominal values \( R_5 = R_{11} = R_{12} = R_{17} = 1\text{k}\Omega, \) \( R_{21} = 1\text{M}\Omega, \) \( R_{18} = 500\text{k}\Omega, \) \( R_8 = 2\text{M}\Omega, \) \( R_3 = R_6 = R_7 = R_9 = 40\text{k}\Omega, \) \( R_2 = R_{14} = 20\text{k}\Omega, \) \( R_1 = R_4 = R_{10} = R_{13} = R_{15} = R_{16} = R_{19} = R_{20} = R_{22} = 100\text{k}\Omega. \) To make a standard mathematical operation, we use cheap operational amplifier TL084. The core of this conception is four-channel four-quadrant analog multiplier MLT04 with transfer function \( W = 0.4XY, \) where \( X \) and \( Y \) are high impedance input nodes. Note that only three basic building blocks are necessary for design the oscillator: inverting integrator, summing amplifier and multiplication cell. This allows us to construct final network directly from the differential equations, as it is demonstrated in Fig. 1. The supply voltage is symmetrical \( \pm 15\text{V} \) and from this voltage \( \pm 5\text{V} \) is derived (on-board) for the proper function of the multiplier. Several bifurcation sequences can be studied simply by independent adjusting of the parameter \( A, D \) or a product of \( AC. \)
Fig. 1: Complete network representing the modified Lorenz system.

Fig. 2: Physical implementation of the chaotic oscillator.
3. Laboratory experiments

For this purpose, digital oscilloscope HP54603B was used. The main importance of the presented dynamical system is in the shape of its chaotic attractors, since it is definitely not familiar to any other known chaotic attractors. The gallery of oscilloscope screenshots is shown through Fig. 3, Fig. 4, Fig. 5 (coexistence of two nearby chaotic attractors if $A=0.1$, $B=4$, $C=14$, $D=0.08$), Fig. 6 (single-folded topology if $A=0.9$, $B=4$, $C=9.9$, $D=1$) and Fig. 7.

![Fig. 3: First type of the chaotic attractor, all plane projections.](image1)

![Fig. 4: Second type of the chaotic attractor, all plane projections.](image2)

![Fig. 5: Third type of the chaotic attractor, all plane projections.](image3)

![Fig. 6: Fourth type of the chaotic attractor, all plane projections.](image4)
4. Conclusion

This paper shows the new and still unpublished experimental results obtained during measurement of the chaotic oscillator. It is easy to verify that the oscilloscope screenshots and numerical integration of the state space trajectory looks like the same. Thus, it is worth nothing that also PSpice circuit simulator provide us with the same results. The period-doubling bifurcation has been observed, so that our oscillator can generate periodic or multi-periodic signals. During the numerical analysis of (1) for various combinations of parameters, we notice possibility of switching $B \rightarrow B$ and $D \rightarrow D$. This was also confirmed experimentally in the early stages of the oscillator design.

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References