Experimental Study of the Sampled Labyrinth Chaos

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Abstract. *In this paper, some new numerical as well as experimental results connected with the so-called labyrinth chaos are presented. This very unusual chaotic motion can be generated by mathematical model involving the scalar goniometrical functions which makes a three-dimensional autonomous dynamical system strongly nonlinear. Final circuitry implementation with analog core and digital parts can be used for modeling Brownian motion.*

From the viewpoint of generating chaotic motion by some electronic circuit, first step is to solve problems associated with the two-port nonlinear transfer functions synthesis. In the case of labyrinth chaos the finite dynamical range of the input variables introduced by the used active elements usually limits the performance greatly, similarly as it holds for the multi-grid spiral attractors. This paper shows an elegant way how to remove these obstacles by using universal multiple-port with internal digital signal processing.

Keywords

Analog oscillator, labyrinth chaos, signal processing, Lyapunov exponents, sampled dynamics.

1. Introduction

A huge pile of the journal articles solving problems with chaos and other types of complex behavior have been published from its discovery about forty years ago. The reliable parts of these publications are about methods how to create chaotic waveform by an oscillator. For any circuit structure and purpose, having certain knowledge about the basic properties of the generated waveforms in time and frequency domain will play a crucial role. Understanding a complex dynamics is useful also from the theoretical point of view because chaos can be considered as the universal phenomenon due to the usual normalization of the internal system parameters and arbitrary interpretation of the state variables. That is the reason why chaos has been experimentally confirmed and reported in many distinct scientific fields [1], for example in analog, digital and power electronics, optics, Newtonian mechanics, weather circulation, chemical reactions, biology, neural networks, traffic, population growth and evolution, economy, etc.

It is well known that the most straightforward method for modeling motion of autonomous or driven dynamical systems is to design the corresponding electronic circuit. Recently it turns out that even Brownian like motion [2] and can be successfully approximated by using mathematical model of the dynamical system with cyclically symmetrical vector field and periodical feedback function. This paper is organized as follows. The next section brings a brief overview on the typical labyrinth chaos dynamics [3], its sampled and switched modification, including numerical integrations and the largest Ljapunov exponent (LE) calculations for each system. The numerical tests of the solution sensitive to the small changes of the initial conditions can be considered as essential part of the pre-study for the circuitry realization. Section 3 deals with the troubles of the mixed mode circuit synthesis. Section 4 fully covers the area of the experimental verification of the final analog oscillator. At the end some future perspectives covering the dynamical system simulation, mixed mode circuit design approach and digital two-port limitations are provided.

2. Original Dynamics

Assume a following general class of the third-order autonomous deterministic dynamical systems with smooth cyclically symmetrical vector field of the form

$$\mathrm{d}x/\mathrm{d}t = f(x,y,z), \quad \mathrm{d}y/\mathrm{d}t = f(y,z,x), \quad \mathrm{d}z/\mathrm{d}t = f(z,x,y), \tag{1}$$

capable to produce chaotic motion for many choices of the nonlinear function *f*. The labyrinth chaos (LCH) can be observed in the system

$$f(x,y,z) = -ax \pm \sin(by), \tag{2}$$

and

$$f(x,y,z) = -ax \pm \cos(by). \tag{3}$$

Standard mathematical tools such as linear algebra can help us to find behavior of linearized system near it´s equilibria. Deep analysis for the sine function and the particular case $b=1$ is already done in [4]. The number and position of the fixed points depends on the values of the both parameters. Note that if parameter $a = 0$ and $b > 0$ the system (2) or (3) turns into the conservative, state space volume preserving one. Low dissipation value has immediate consequence in that the strange attractor becomes large and sorely limited in the case of real circuit with a certain range of the power supply voltage. Thus there is always a positive dissipation factor represented by a single resistor connected in parallel with each inverting integrator. There are always the lossy elements in the real circuit adding the desired terms into the state matrix. On the contrary to conventional circuits these error terms have positive effect on global dynamics.

2.1 Numerical Analysis

In the case of chaotic systems it is always impossible to obtain an analytic solution. Because of this, to analyze a given system we are restricted to use numerical approach, i.e. algorithms based on the numerical integration process. For graphs of the largest LE as well as the state space attractor visualization program Mathcad and build-in fourth-order Runge-Kutta method has been utilized with final time t_{max}= 1000, number of steps N = 10^5 leading to step size Δ_t= 0.01 and initial conditions $\mathbf{x}_0$=(0.1 0 0)^T. To avoid errors, also fourth-order Runge-Kutta method has been used and compared with fixed step method. The fixed step method proved to be sufficiently precise and fast. A spectrum of three LEs has been calculated with extra precision Δ_a= 0.01 and Δ_b= 0.01 using the averaging principle and Gram-Smith orthogonalization technique (performed repeatedly after few calculation steps to prevent the vector base being deformated under the flow) inside main cycle loop. It is worth nothing that each LE is a real number and one LE converges to zero [5]. The exponential divergence of the neighborhood trajectories is responsible for sensitivity to the initial conditions as well as for positive value of the largest LE. This idea is visible by the contour plots of the largest LE as function of the parameter *a* and *b* shown in Fig. 1 for the original LCH with sine and cosine function, in Fig. 2 for negative sine and cosine function respectively. The color scale from dark blue through green toward white represents the range of $LE_{max} \in$ <-1; 3.231>. The interesting results follow directly from the observation of these graphs, especially its mutual similarity. This property leads to the proposition that LCH can be generated by many forms of the periodical feedback functions. The still unanswered question is the necessity to use smooth nonlinear functions. It has been verified numerically that some sort of LCH is preserved also in the case of sign or saw-tooth type of feedback functions. This can be essential from the viewpoint of practical implementation of LCH as an electronic circuit since periodical signum transfer curve can be simply realized by a cascade connection of the comparators [6]. The gallery of the typical trajectories wandering the state space is shown in Fig. 3 and Fig. 4 for sine and cosine feedback function respectively. Similarly Fig. 5 and Fig. 6 give an idea how the state space attractors look like if the negative sine and cosine function is utilized. Note that the global behavior of the oscillator is qualitatively the same. It is evident that the trace of the Jacobi matrix given uniquely by parameter *a* affects the size of the state space attractor.

To preserve feasibility of LCH this parameter should be larger than 0.2. In spite of the Monge projection on to *xy* plane, topographic color scale changes colors in the state variable *z* direction giving better idea about the full three dimensional nature of the individual state space attractors.

There are several concepts how to extract LE from the time series, i.e. using a data set measured by digital oscilloscope. These results are not provided since these methods [7] are highly inaccurate and almost useless.

3. Sampled Dynamics

Assume that the nonlinear transfer function is realized by using some sort of the digital circuit. The input variables $x(t)$, $y(t)$ and $z(t)$ need to be sampled with the sampling frequency much higher than the largest natural frequency component of original chaotic motion. Having the record of samples denoted as $x(nT_S)$, $y(nT_S)$ and $z(nT_S)$ there are two distinct cases of the further signal processing. The first case deals with the possibility of the independent three-channel digital signal processing such that there is no need to use a multiplexer and demultiplexer at the input and output of the digital two-port respectively. The second case assumes a single channel digital processing stage with synchronized multiplexer and demultiplexer added into a signal path. To date many high-tech processors (from the viewpoint of real time computational performance) are commercially available [8]. Generally there are serious problems with a quite wide dynamical range of the useful analog signal, especially in the case of the labyrinth chaos with low dissipation factor. To overcome the drawback of adding circuits for signal compression and expansion integrated circuit TLC2574 has been chosen as 10 bit A/D converter. Using manufacturer data this device has the dynamical range ±10 V, theoretical speed is 200 KSPS, and clock frequency is 25 MHz and SPI bus for interconnection with MCU. For inverse D/A conversion four-channel DAC8734 with output dynamics ±16 V has been utilized. The core engine in the digital part is MCU denoted as STM32F107 with 512KB of internal memory. In this case the clock frequency is 72 MHz and thus offers up to 90 MIPS. Input 12 bits A/D converter is fast enough but needs to compress the input voltage below the granted range. Before the first run of the MCU particular application the basic mathematical functions benchmark has been started. The final program has been implemented in C/C++ language in KEIL uVision V3.90 freeware environment. The most time-consuming operations are evaluations of the trigonometric functions. Using all three channels simultaneously the maximum signal frequency is about 231 kHz.

3.1 Numerical Analysis

Numerical integration with the set of parameters given above has been utilized for normalized sampling frequency $f_S = 100$ and several different horizontal and vertical resolutions of the digital transfer function six-port. The resulting state space trajectories are shown in Fig. 7 and Fig. 8 for the sine function. The integration method and associated parameters were exactly the same as hold for the numerical analysis with the smooth functions.

4. Circuit Measurement

The main advantage of the proposed circuit conception is in the implementation of the linear part of the vector field. Following the rules for synthesis based on the integrator block schematic [9] it consists only of three lossy inverting integrators connected to the digital nonlinear two-port as shown in Fig. 11. Thus frequency renormalization is an easy and straightforward process, namely identical changes of all integration constants simultaneously. In the experimental setup values $C = 100$ nF and $R = 22$ kΩ have been used. The resistors R_a represent the LCH parameter a, i.e. dissipation losses in the mathematical model as well as in the real circuit. It seems, in practice, output of the digital part needs to be a smooth continuous time signal, at least approximately. The selected plane projections of the state space attractors provided by sine and cosine function are given in Fig. 9 and Fig. 10 respectively. The concrete values of the resistors R_a and parameter b adjusted in the digital part of the circuit are not provided since there is always some uncertainty in the setting. By careful observation the same routing to chaos scenario has been confirmed in the dynamical system with smooth and sampled nonlinear function. Note there is a very good relationship between theoretical expectations and the experimental measurements for both cases of feedback functions. Unfortunately minor problems can occur during the process of calculating the spectrum of one-dimensional LEs. This is because the partial derivatives of the sampled function take only extreme values: zero and positive and negative infinity. This difficulty can be removed by using Fourier series expansion of piecewise-constant sine or cosine function.

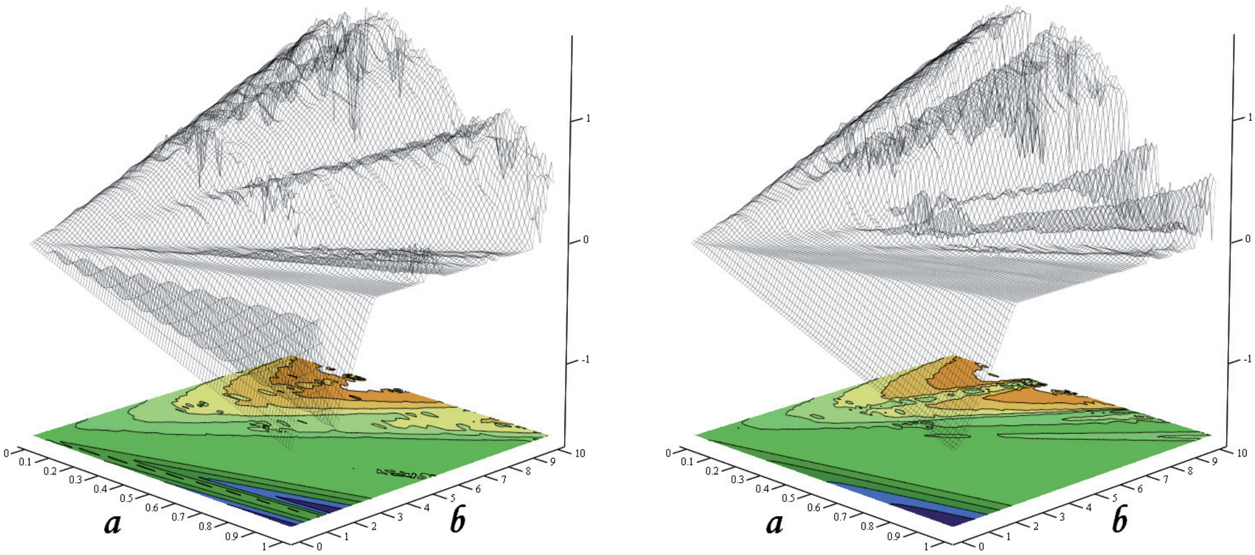

Fig. 1. Contour plot of the largest $LE_{max}=\Psi(a,b)$ of the smooth LCH with sine function (left) and cosine function (right).

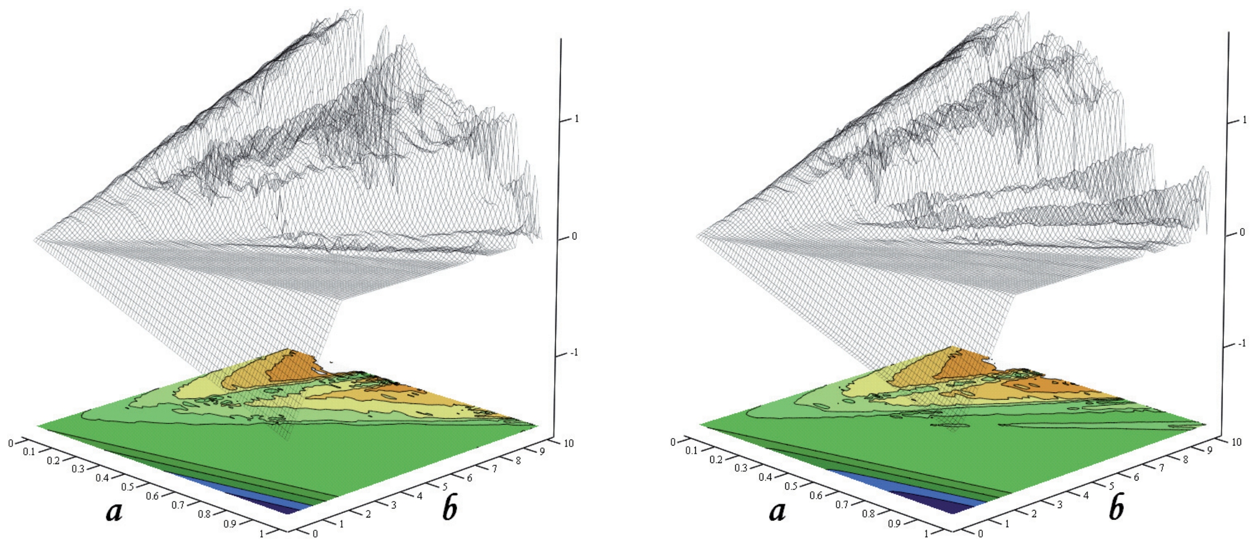

Fig. 2. Contour plot of the largest $LE_{max}=\Psi(a,b)$ of the smooth LCH with negative sine function (left) and negative cosine function (right).

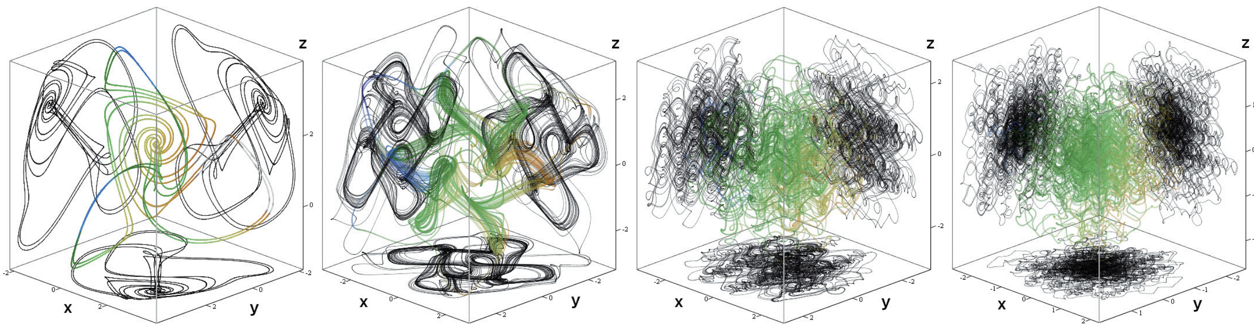

Fig. 3. Individual LCH with sine function in the state space, parameter sets [*a*=0.2 *b*=1], [*a*=0.2 *b*=2], [*a*=0.2 *b*=5], [*a*=0.2 *b*=10].

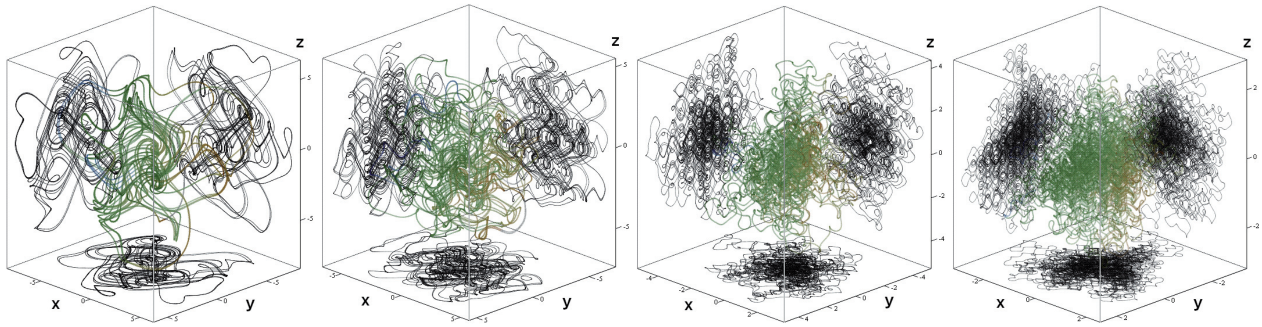

Fig. 4. Individual LCH attractors with cosine function, following parameter sets [*a*=0.1 *b*=1], [*a*=0.1 *b*=2], [*a*=0.1 *b*=5], [*a*=0.1 *b*=10].

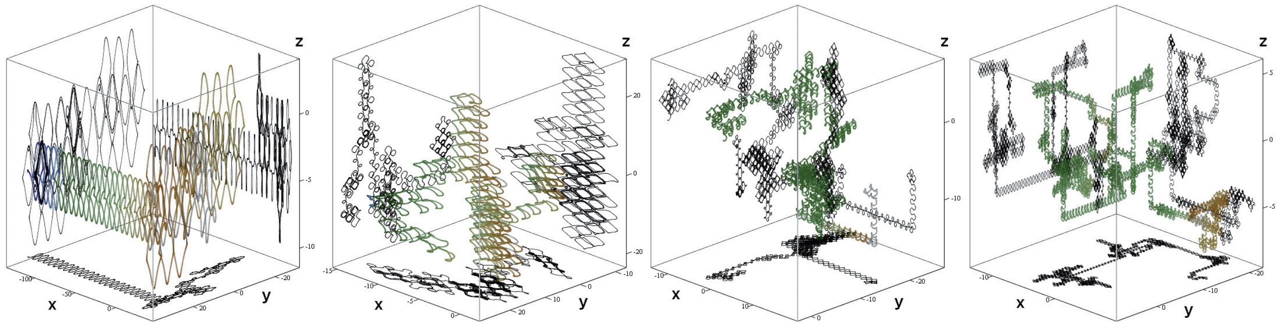

Fig. 5. Individual LCH attractors with negative sine function, parameter sets [*a*=0.1 *b*=1], [*a*=0.1 *b*=2], [*a*=0.1 *b*=5], [*a*=0.1 *b*=10].

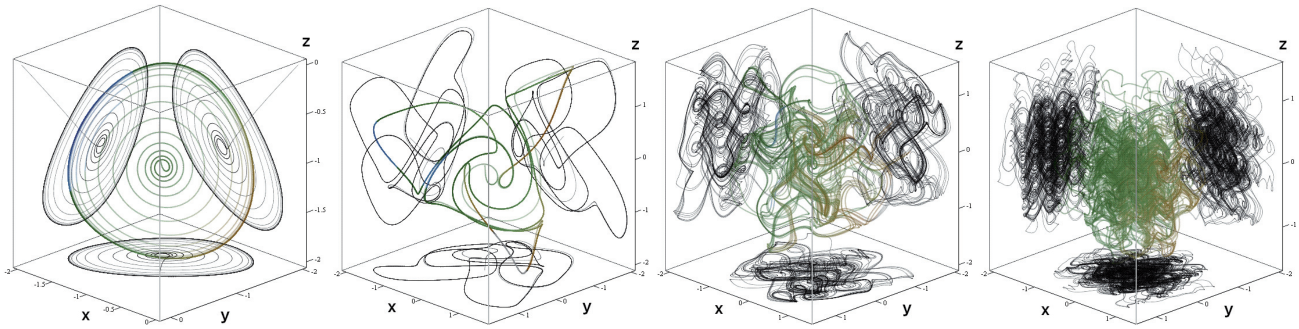

Fig. 6. Individual LCH attractors with negative cosine function, parameter sets [*a*=0.4 *b*=1], [*a*=0.4 *b*=2], [*a*=0.4 *b*=5], [*a*=0.4 *b*=10].

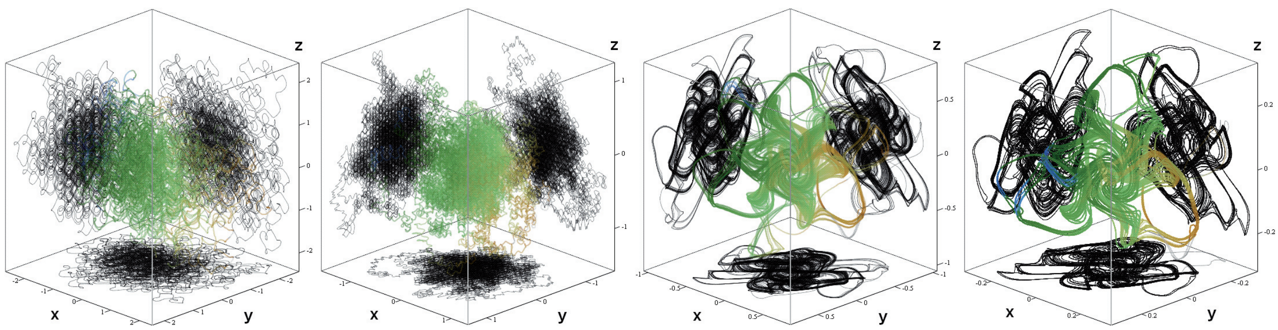

Fig. 7. LCH attractors with sine function using horizontal and vertical 7 bit sampling [*a*=0.1 *b*=1], [*a*=0.1 *b*=5], [*a*=0.9 *b*=2], [*a*=2.2 b=5].

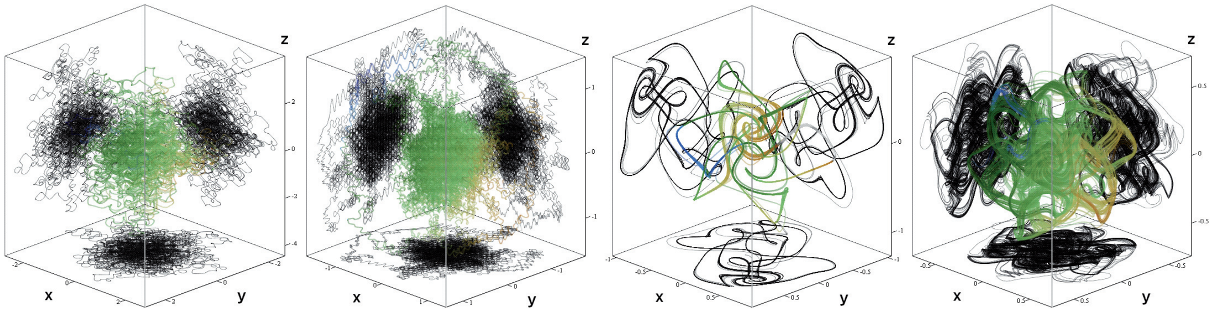

Fig. 8. LCH attractors with sine function using horizontal and vertical 8 bit sampling [a=0.1 b=1], [a=0.1 b=5], [a=0.8 b=1], [a=1 b=3].

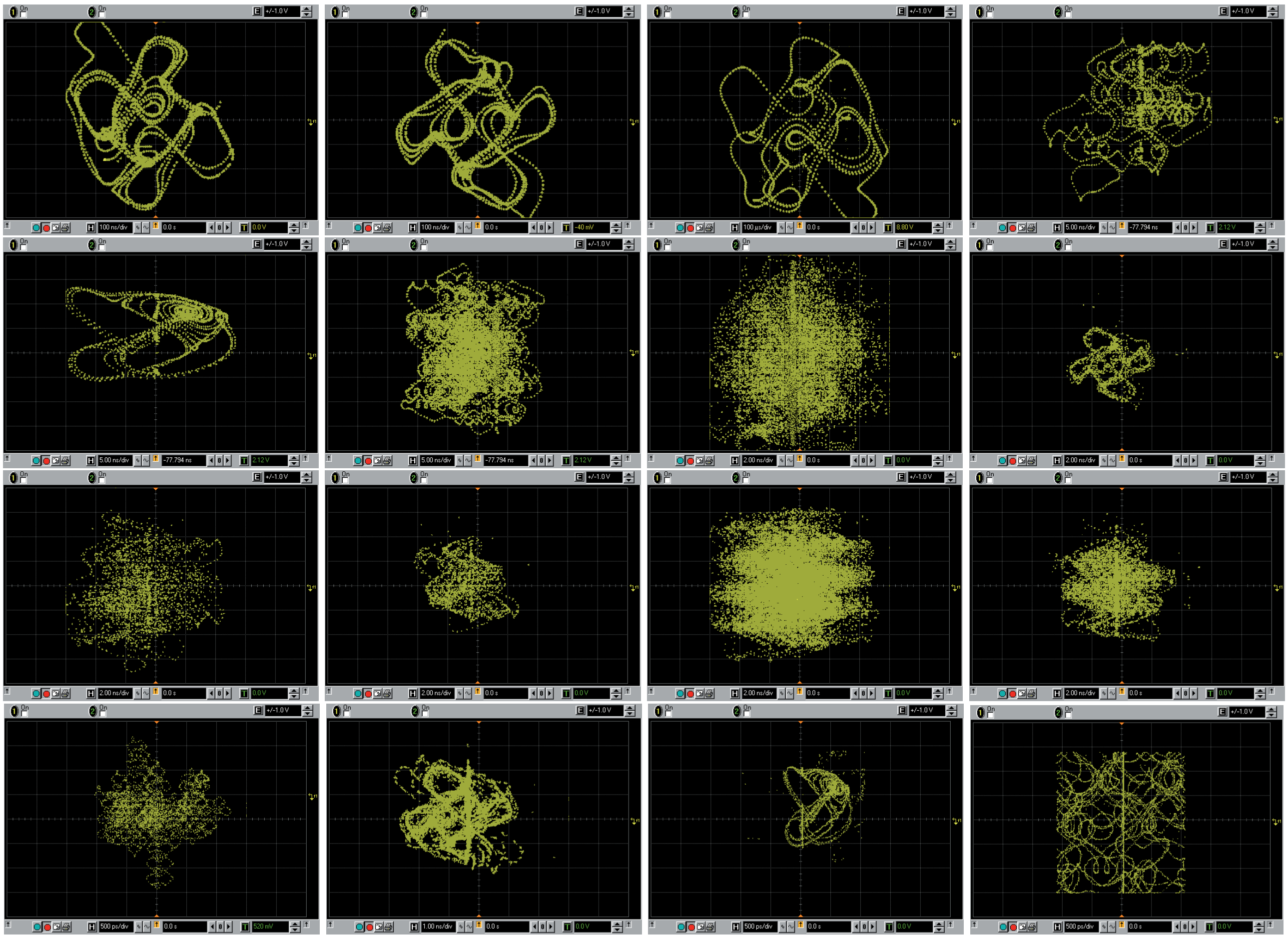

Fig. 9. Screenshots of the measured plane projections of the LCH with sine transfer function, digital oscilloscope Agilent Infiniium.

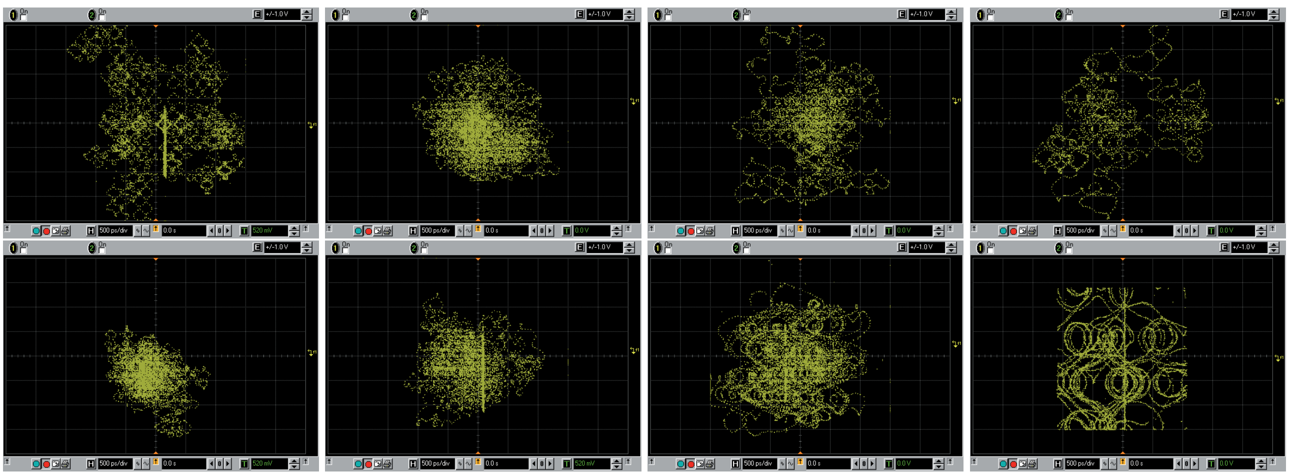

Fig. 10. Screenshots of the measured plane projections of the LCH with sine transfer function, digital oscilloscope Agilent Infiniium.

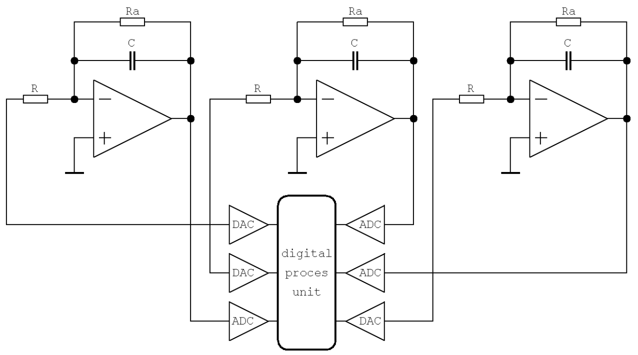

Fig. 11. Principal configuration of LCH mixed mode oscillator.

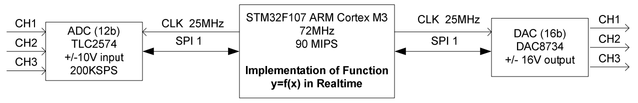

Fig. 12. Detailed block schematic of the digital six-port circuit.

5. Conclusion

The main impact of this work is in that, referring to the best knowledge of the authors, this is the first successful circuit implementation of the LCH generator. Thank to the digital part of the final network shown in Fig. 12 there is a large variability of the proposed oscillator. Thus it is optimal experimental tool for studying, however greatly simplified, the molecular dynamics as well as the potential interactions between such microscopic particles.

In this article the influence of sampling process on the LCH has been numerically studied. It follows from the particular results given above that the nature of LCH is preserved if the sampling ratio is high enough. Due to the large scale of some state space attractors and limited dynamical range of the digital elements some parameters of the mathematical model are not suitable for practical implementation. If interested in more details, readers are encouraged to contact the corresponding author. Useful information can be also found for example in [10].

The basic properties of LCH examined by means of numerical analysis have been discussed. Huge number of the simulations reveals some cases of simpler nonlinear functions with the chaotic behavior like piecewise-linear (with the same positions of fixed points) or piecewise-constant approximations of smooth sine and cosine function respectively. It is likely the first step to implement a fully analog LCH oscillator. It is also the way how to avoid a construction of the transfer function with goniometrical shape by a cascade connection of many analog multipliers. This is possible due to the approximation of sine or cosine functions by a power row. This approach is suggested in [11] but the final oscillator will be affected by noise since the approximation polynomial has very small coefficients coupled with higher powers of input variable. The general overview and concrete example how to practically realize nonlinear dynamical systems can be found in [12] and [13] respectively. The selected topics from the nonlinear dynamics and chaos theory suitable for further research are provided in [14] and [15].

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