Measurement of Spatial Coherence of Light Propagating in a Turbulent Atmosphere

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Abstract. *A lot of issues have to be taken into account when designing a reliable free space optical communication link. Among these are e.g.,beam wander, fluctuation of optical intensity and loss of spatial coherence that are caused by atmospheric turbulence. This paper presents experimental measurements of spatial coherence of a laser beam. The experimental setup is based on Young's double pinhole experiment. Fringe patterns under atmospheric turbulence for four different pinhole separations are presented. From these fringe patterns, visibility is determined and the coherence radius is estimated.*

Keywords

Atmospheric turbulence, spatial coherence, Young's experiment, fringe pattern, coherence radius.

1. Introduction

Free space optical links use optical carriers with one or several wavelength channels for transmitting data. The optical power is focused into one or several optical beams. The main advantages of free space optical links are a license-free band, electromagnetic compatibility and a high data rate [1]. The quality of these networks is highly dependent on weather conditions because the atmosphere is a transmission medium. The study of effects associated with propagation of light through this medium is a widely solved task in many researches [2–5]; some of them are focused on measuring spatial coherence [6–8]. In this paper we present the measurement of spatial coherence of a laser beam by using Young's double pinhole interferometer under different degrees of turbulence.

2. Atmospheric transmission medium

The three phenomena that affect laser beam propagation through the atmosphere are scattering, absorption and atmospheric turbulence [9]. A laser beam can quickly lose a part of its energy and, moreover, this loss can lead to beam quality degradation. Signal power is randomly modulated and phase front is distorted [10]. Atmospheric turbulence is one of the most significant phenomena in atmospheric transmission medium. Temperature differences between the earth's surface and the atmosphere with wind variations create local unstable air masses which are broken up into turbulent eddies. The size of these eddies varies from millimeters to hundreds of meters. The characteristics of the fluctuations may be expressed by a structure function of refractive index $D_n(\mathbf{r})$

$$D_n(\mathbf{r}_1,\mathbf{r}_2) = \langle [n(\mathbf{r}_1) - n(\mathbf{r}_2)]^2 \rangle \tag{1}$$

where $n(\mathbf{r}_1)$ is refractive index at point $\mathbf{r}_1$ and $n(\mathbf{r}_2)$ is refractive index at point $\mathbf{r}_2$. Sharp brackets denote an ensemble average. For homogeneous and isotropic turbulences the dependency on distance r can be written as

$$D_n(\mathbf{r}) = C_n^2 r^{2/3}, \quad l_0 \ll r \ll L_0 \tag{2}$$

and

$$D_n(\mathbf{r}) = C_n^2 l_0^{-4/3} r^2, \quad r \ll l_0 \tag{3}$$

where l_0 is inner scale (millimeters) and L_0 is outer scale (hundreds of meters) of turbulences [9].

The refractive index structure parameter C_n^2 constitutes a measure of the turbulence. Typically, the values of C_n^2 range from $10^{-16}\,m^{-2/3}$ for weak turbulence to $10^{-12}\,m^{-2/3}$ for strong turbulence.

Atmospheric turbulence leads to irradiance fluctuations, beam spreading and loss of spatial coherence of a laser beam [9].

3. Spatial coherence

As known, a laser is a source of light with high temporal and spatial coherence. Spatial coherence is the correlation of the electric fields at two different positions ($\mathbf{r}_1$, $\mathbf{r}_2$) on the same wave front. Temporal coherence is the correlation between the fields at two different times (t_1, $t_2 = t_1 + \tau$) in the same wave train. The mutual coherence function Γ combines both spatial and temporal characteristics in one single term. It is defined as

$$\Gamma(\mathbf{r}_1,\mathbf{r}_2,\tau) = \langle U(\mathbf{r}_1, t+\tau) U^*(\mathbf{r}_2, t) \rangle \tag{4}$$

where $U(\mathbf{r}_1,t)$ is the complex electric field at position $\mathbf{r}_1$ and time t, τ is time delay [11].

The principle of Young's double slit experiment is shown in Fig. 1. Two pinholes are illuminated by a laser beam. Intensities I_1 and I_2 from the pinholes P_1 and P_2 interfere at the point P at the plane π.

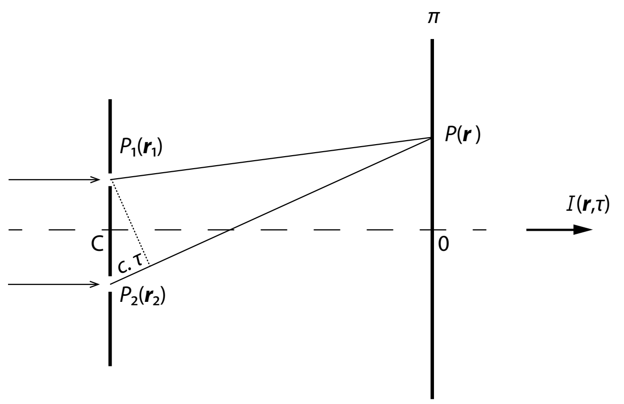

Fig. 1. Young's double slit experimental arrangement.

The normalized mutual coherence function γ_{12} is given with relation (5) and is called the complex degree of coherence [11]

$$\gamma_{12} = \frac{\Gamma(\mathbf{r}_1,\mathbf{r}_2,\tau))}{\sqrt{I_1 I_2}}. \quad (5)$$

Intensity distribution at π plane is described by the equation (6)

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\gamma_{12}(\tau)cos(\varphi_{12}(\tau)). \quad (6)$$

where φ_{12} is the phase. Visibility V of the interference pattern can be defined by the formula [12]

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \quad (7)$$

where I_{max} is the maximum value and I_{min} is the minimal value of the optical intensity I at the π plane. If the maximal and the minimal value of the optical intensity from relation (6) is deduced and these values are put into equation (7) we obtain

$$V = \frac{2\sqrt{I_1 I_2}\gamma_{12}(\tau)}{I_1 + I_2}. \quad (8)$$

An expression for the complex degree of coherence γ_{12} (9) comes out from relationship (8)

$$\gamma_{12}(\tau) = \frac{V(I_1 + I_2)}{2\sqrt{I_1 I_2}}. \quad (9)$$

From equation (9) is clear that if the intensity I_1 and I_2 are equal, then the absolute value of γ_{12} is identical to the visibility V of the fringe caused by the interference of two wave fronts [11]. The value of γ_{12} indicates the degree of the laser beam coherence for a given spectral linewidth, pinhole separation and the state of the atmosphere. Loss of spatial coherence can be deduced from a complex degree of coherence. Visibility of interference fringe is reduced with increasing degree of atmospheric turbulences and separation of pinholes. Loss of spatial coherence limits the effective aperture size of heterodyne detection optical receivers [9].

The coherence radius is useful for determining the size of the receiver aperture through a process called aperture averaging and also for determining the separation distance of detectors in a multiple receiver system [13]. The coherence radius is defined like separation distance at which the modulus of the complex degree of coherence falls to $1/e$ [9]. In Ref. [14] spatial coherence radius for the infinity plane wave ρ_{pl} and spherical wave ρ_{sp} is given by

$$\rho_{pl} = (1.46C_n^2 k^2 L)^{-3/5}, \quad l_0 \ll \rho_{pl} \ll L_0, \quad (10)$$

$$\rho_{sp} = (0.55C_n^2 k^2 L)^{-3/5}, \quad l_0 \ll \rho_{sp} \ll L_0 \quad (11)$$

where k is the wave number and L is the distance between the optical transmitter and receiver. The meaning of parameters l_0 and L_0 is mentioned in the previous chapter. For a collimated Gaussian-beam wave, the spatial coherence radius ρ_g is approximated by [14]

$$\rho_g = (0.55C_n^2 k^2 L(a + 0.62\Lambda^{11/6}))^{-3/5}, \quad l_0 \ll \rho_g \ll L_0 \quad (12)$$

where the Fresnel ratio Λ is for a beam width at the receiver plane w defined by

$$\Lambda = \frac{2L}{kw^2}. \quad (13)$$

Parameter a is defined by

$$a = \frac{1 - \Theta^{8/3}}{1 - \Theta}, \quad \Theta \geq 0 \quad (14)$$

where Θ is the refractive beam parameter and can be calculated by

$$\Theta = \frac{1}{\left[1 + \left(\frac{2L}{kw_0^2}\right)^2\right]} \quad (15)$$

where w_0 is beam width at the waist.

4. Experimental measurement

In [15] it has been shown that the loss of coherence in a turbulent atmosphere can be experimentally determined by measuring spatial interference in a modified Mach-Zender interferometer. We chose a simpler method based on Young's pinhole experiment.

For the experimental measurement we used a He-Ne laser with a wavelength of 632.8 nm. The beam width at the waist was w_0 = 0.28 mm and at the receiver plane it was w = 1.06 mm. Two heaters with an effective diameter of 185 mm were used for generating a turbulent atmosphere. The laser beam passed 100 mm above the heaters. For each temperature level the relative variance of optical power σ_P^2

was measured and the relative variance of optical intensity σ_I^2 was calculated. The ratio of the relative variance of optical power and the relative variance of optical intensity is given by [5]

$$\frac{\sigma_P^2}{\sigma_I^2} = \left(1 + 1.062\frac{kD^2}{4L}\right)^{-7/8}. \tag{16}$$

This ratio is determined by the diameter of the receiving aperture D = 0.2 mm, the link distance L = 1.9 m and the wave number k. From the calculated value of relative optical intensity variance σ_I^2 we estimated the C_n^2 parameter according to Rytov variance [9]

$$\sigma_I^2 = 1.23 C_n^2 k^{7/6} L^{11/6}. \tag{17}$$

The beam passed through two pinholes with a diameter of 0.5 mm each. The pinhole separation d was set to 0.5 mm, 1 mm, 1.5 mm and 2 mm. The beam was locked exactly between the two pinholes in the vertical and horizontal plane. At the receiver plane we obtained a fringe pattern caused by the interference. Optical intensity was scanned with the CCD camera SP620U Ophir Spiricon. The distance between the laser source and the camera was set to 1.9 m.

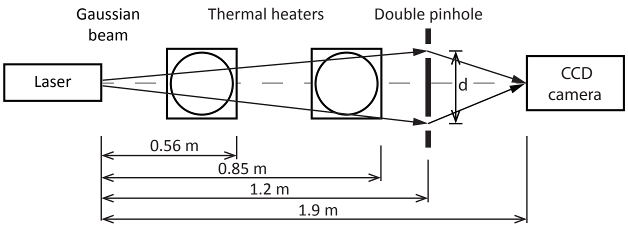

Fig. 2. Experimental setup for spatial coherence measurement.

Furthermore, from the captured fringe patterns we calculated visibility according to relation (7). Because the fringe patterns were unstable due to atmospheric turbulence, visibility was calculated from the 40 captured fringe patterns which were averaged. The observed fringe patterns for all pinhole separations are depicted in Fig.3 - Fig.6.

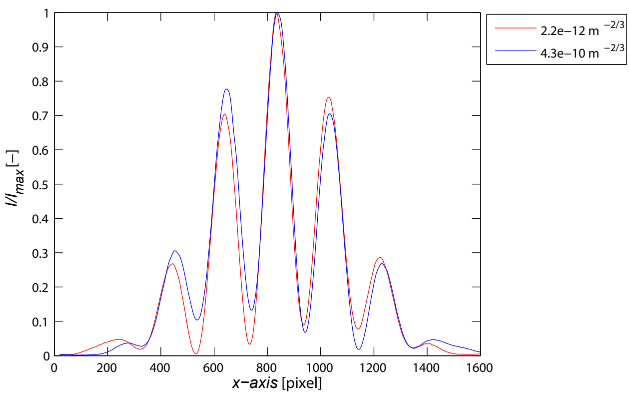

Fig. 3. Observed horizontal fringe patterns for pinhole separation d = 0.5 mm under different degrees of turbulent atmosphere.

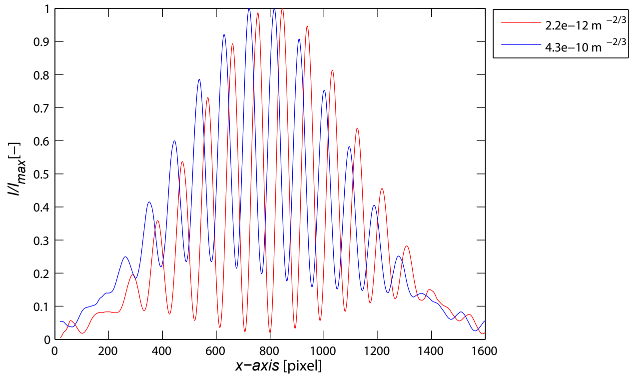

Fig. 4. Observed horizontal fringe patterns for pinhole separation d = 1 mm under different degrees of turbulent atmosphere.

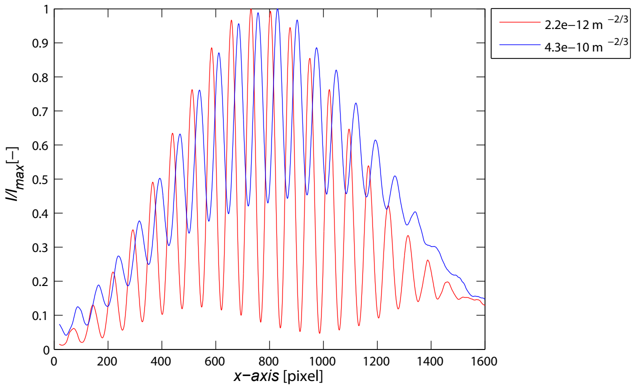

Fig. 5. Observed horizontal fringe patterns for pinhole separation d = 1.5 mm under different degrees of turbulent atmosphere.

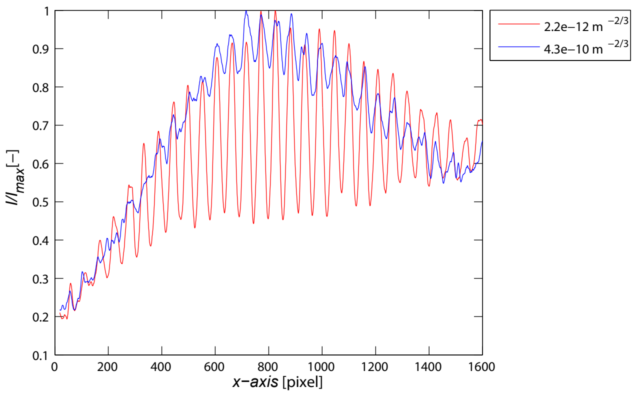

Fig. 6. Observed horizontal fringe patterns for pinhole separation d = 2 mm under different degrees of turbulent atmosphere.

In Fig.7 the dependency of visibility on the degree of turbulence is shown. Measured visibility was almost constant for slit separation d = 0.5 mm due to small time delay τ. The loss of spatial coherence is notable for pinhole separation bigger than d = 1 mm. For pinholes 1.5 mm apart, visibility at minimal degree of turbulence reaches a value of 0.87. If temperature on heaters were set to maximum, visibility would fall to 0.4. It can be observed that measured optical power in the interference fringes for slit separation d = 2 mm was so small (relative to beam halfwidth) that measurement of the visibility was on the limit of sensitivity of the CCD camera.

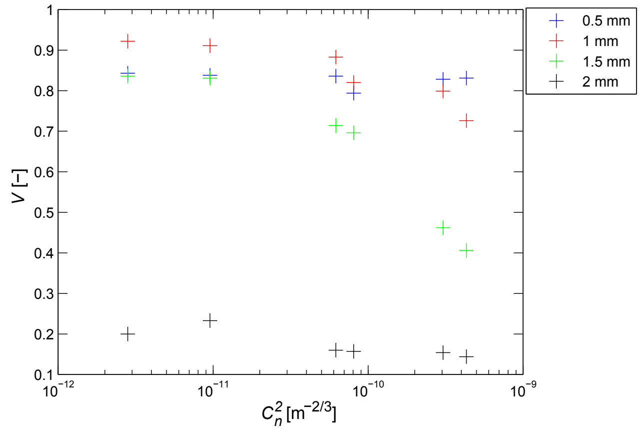

Fig. 7. Dependency of interference fringe visibility on refractive index structure parameter.

Dependency of fringe pattern visibility on pinhole separation is shown in Fig.8. From this figure we estimated coherence radius ρ_0 at the receiver plane.

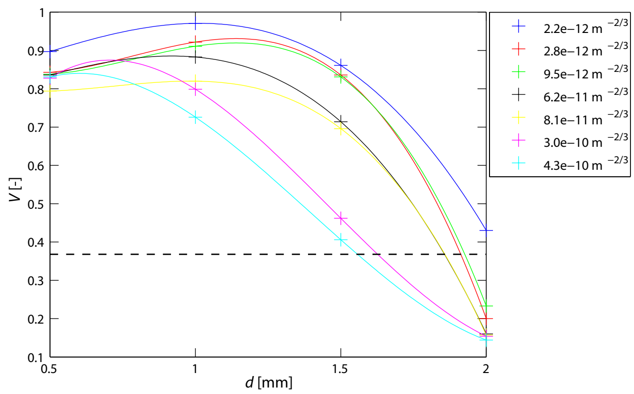

Fig. 8. Dependency of interference fringe visibility on slit separation.

The coherence radii for different refractive index structure parameters are listed in Tab. 1.

Structure parameter C_n^2 $[m^{-2/3}]$	**Coherence radius** ρ_0 $[mm]$
$2.20.10^{-12}$	2.00
$2.80.10^{-12}$	1.87
$9.54.10^{-12}$	1.88
$6.19.10^{-11}$	1.81
$8.07.10^{-11}$	1.80
$3.04.10^{-10}$	1.65
$4.31.10^{-10}$	1.57

Tab. 1. Coherence radius for different degrees of turbulence.

Theoretical values of the coherence radius for different refractive structure index parameters were determined from theoretical expressions of plane, spherical and Gaussian-beam wave. Thereafter, these values were compared with measured values and depicted in Fig. 9.

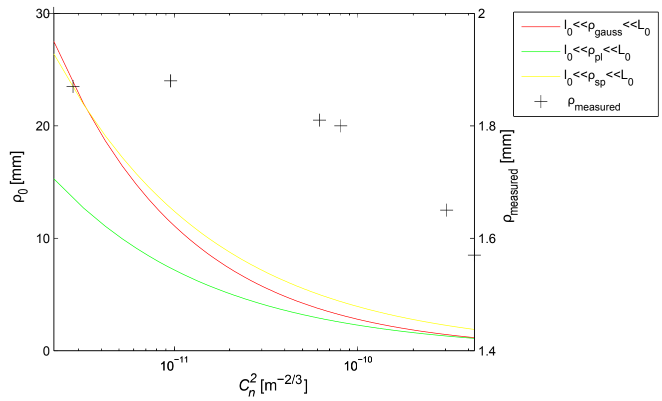

Fig. 9. Dependency of theoretical coherence radius on refractive index structure parameter.

From Fig.9. it is evident that the coherence radius decreases with increasing degree of turbulence. However, measured values (Fig.9.) of spatial coherence radius did not correspond with theoretical models. One of the possible causes of the disagreement between the measured and calculated results could be incorrectly determined C_n^2 parameters. Accurate estimation of C_n^2 parameter can be an issue and for this reason an experiment is being prepared. In this experiment the C_n^2 parameter will be measured with independent different methods simultaneously.

5. Conclusion

The paper presents an experimental measurement of spatial coherence of a laser beam in turbulent media. The experimental setup was based on Young's double pinhole experiment, which is simpler than the generally used method based on the modified Mach-Zender interferometer. Horizontal fringe patterns for defined pinhole separation were recorded. The coherence radius for different degrees of atmospheric turbulence was estimated from visibility of the fringe patterns caused by interference. From this measurement it is clear that increasing degree of atmospheric turbulence leads to a loss of spatial coherence of the laser beam. The observed characteristic is non-linear. This fact should be taken into account in the experiments and measurements employing coherent laser beam with potential occurrence of atmospheric turbulence.

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