2.1 Diffraction on a planar absorbing object

Advanced theory

Assume a planar object, which is perpendicular to the line *VP* (wave source - observation point) and which is partially covered by a perfectly absorbing matter. The rest of the plane is free, and therefore, waves can past through this area to the half-space containing the observation point *P*. Computing field intensity in *P* is our task. Except of the marginal case (all the plane is free), we deal with the diffraction task.

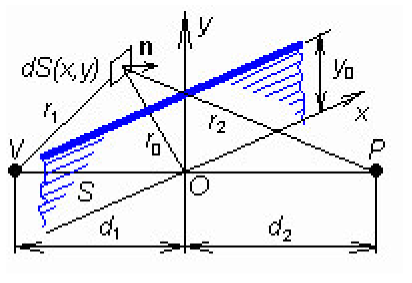

Fig. 2.1B.1 *Half-plane diffraction*

A classical solution of the above-formulated task was provided by Fresnel (*Fresnel diffraction*). Fresnel assumed the planar obstacle (field intensity of its reverse side) as a wave source for the half-plane containing the observation point *P*. Hence, Fresnel understood the planar obstacle as a large planar antenna. Next, Fresnel expected the intensity on the reverse side being zero in points covered by the absorbing matter and being non-zero (uninfluenced by the obstacle) in points above the obstacle edge. Both the assumptions are inaccurate but the reality usually approaches them. Thanks to this conception, the diffraction task is well solvable as two independent tasks: wave propagation from the source to the obstacle plane and wave propagation from free regions of the plane to the observation point. That way, the obstacle was removed pro computations and the result is influenced by the shape of free regions only.

In the basic theory, Fresnel solution for the half-plane obstacle (fig. **2.1B.1**) was briefly presented.

Since spherical wave propagates from the source *V*, field intensity on the obstacle plain is

$$E^{(S)} = C\,\frac{e^{-jkr_1}}{r_1}, \qquad (2.1B.1)$$

where k is wave number, C is a constant dependent on a radiated power and a directivity factor of the transmitting antenna, and where the distance r_1 is given by $r_1 = \sqrt{d_1^2 + x^2 + y^2}$. Symbols x, y denote coordinates of a point on the obstacle plane. Every point (x, y), or every element $dS = dxdy$ is left-illuminated by the intensity (2.1B.1) and it becomes (according to Huygens principle) the wave source for the right-hand half-space. It contributes to the intensity in *P* by $dE^{(P)} = \frac{j}{\lambda} E^{(S)} \cos(n, r)\,\frac{e^{-jkr}}{r}\,dS$.

Integrating all the contributions of the free part of the obstacle plane (i.e. $-\infty < x < +\infty$, $y_0 < y < +\infty$) we yield the final intensity in *P*

$$E^{(P)} = \frac{j}{\lambda} \int_{-\infty}^{+\infty} \int_{y_0}^{+\infty} E^{(S)} \cos(n, r_2)\,\frac{e^{-jkr_2}}{r_2}\,dydx. \qquad (2.1B.2)$$

Let us note that changing the shape of the *window* in the obstacle plane changes integration limits only.

In order to finish the computations, Fresnel proposed some simplifications, which are characteristic to his approach. They are mostly based on the assumption that the strongest contribution to the field intensity in *P* is given by elementary sources *dxdy* lying near to the line *VP*, i.e. sources lying on coordinates specified by inequalities $x \ll d_{1,2}$ and $y \ll d_{1,2}$. Then, we can assume $\cos(n,r_2) = 1$, and in denominators $r_1 = d_1$, $r_2 = d_2$. In phase terms, distances r_1 and r_2 have to be expressed more accuratelly

$$r_1 = \sqrt{d_1^2 + x^2 + y^2} = d_1 \sqrt{1 + \left(\frac{x}{d_1}\right)^2 + \left(\frac{y}{d_1}\right)^2} \cong d_1 \left[1 + \frac{1}{2}\left(\frac{x}{d_1}\right)^2 + \frac{1}{2}\left(\frac{y}{d_1}\right)^2\right]. \qquad (2.1B.3)$$

Performing the described rearrangements, we get

$$E^{(P)} = C\frac{j}{\lambda} \int_{y_0}^{\infty} \int_{-\infty}^{\infty} \exp(-jax^2)\exp(-jay^2)\,dxdy \qquad (2.1B.4)$$

where

$$a = \frac{k}{2}\,\frac{d_1 + d_2}{d_1 d_2}. \qquad (2.1B.5)$$

Since in the integrand of (2.1B.4), every term is a function of a single variable, knowledge of the solution of the integral $\int \exp(-jau^2)\,du$, $u = x, y$ is sufficient. Substituting

$$u = v\sqrt{\frac{\pi}{2a}} \qquad (2.1B.6)$$

we transform the above integral to Fresnel integrals

$$\int_0^v \exp\left(-j\frac{\pi}{2}v^2\right)dv = \int_0^v \cos\left(\frac{\pi}{2}v^2\right)dv - j\int_0^v \sin\left(\frac{\pi}{2}v^2\right)dv = C(v) - jS(v). \qquad (2.1B.7)$$

Values of Fresnel integrals $C(v)$ and $S(v)$ are tabularized, are programmed and are graphically expressed as coordinates of the curve depicted in fig. **2.1B.2**. The curve is called klothoidy. Numbers on the curve are argument values v and respective values of integrals $C(v)$ and $S(v)$ are read on axes. The right (upper) branch ends at the point $C(v=\infty) = 0.5$, $S(v=\infty) = 0.5$ and the left branch is symmetric with respect to the origin.

The final solution of the task is of the form

$$E^{(P)} = C\frac{\exp[-jk(d_1+d_2)]}{d_1+d_2}\frac{\sqrt{2}}{4}\exp\left(\frac{j\pi}{4}\right)\{[1-2C(v_0)] - j[1-2S(v_0)]\}, \quad v_0 = y_0\sqrt{\frac{2a}{\pi}}. \qquad (2.1B.8)$$

Dependency of the field intensity in P (behind the obstacle) on the obstacle height y_0 was discussed in the basic theory. We remind here that the obstacle influences the wave propagation even if the direct line VP is not interrupted and that the obstacle can cause even a small increase of intensity (comparing to free-space propagation). If the distance between the obstacle edge and the line VP becomes smaller than one half of the first Fresnel zone then field intensity starts to decrease below the level given by free-space propagation.

Since in the integrand of the integral (2.1B.4), every term is a function of a single variable, the integral can be rewritten as a product of two independent integrals: the first one according to dx, the second one according to dy. The integral according to dx is of infinite limits, its value is independent on the obstacle height and is constant for the given situation. I.e., field intensity in P is proportional to the value of the integral according to y, and therefore, using the substitution (2.1B.6)

$$\int_{v_0}^{\infty} \exp\left(-j\frac{\pi v^2}{2}\right)dv = [C(\infty) - jS(\infty)] - [C(v_0) - jS(v_0)], \quad v_0 = y_0\sqrt{\frac{2a}{\pi}}. \qquad (2.1B.9)$$

The right-hand side of (2.1B.9) is a difference of two complex numbers in a complex plane $C(v)$, $jS(v)$, i.e. in a klothoidy plane in the fig. **2.1B.2**, where the vertical axis plays the role of the imaginary one.

Difference of coordinates of two points in a complex plane equals to the abscissa length from the first point to the second one. Finally, the field intensity in P is proportional to the abscissa length from $[C(v_0), S(v_0)]$ to $[C(\infty), S(\infty)]$ on the klothoidy. The point $[C(\infty), S(\infty)]$ is a point of coordinates 0.5, 0.5 and is fixed. If the upper edge of the obstacle rises to the line VP, the point $[C(v_0), S(v_0)]$ being on the left (down) klothoidy. Klothoidy branch travels along the curve towards the origin and the abscissa length rises and decreases in cyclic way. When $[C(v_0), S(v_0)]$ leaves the last spire of the left branch, oscillations stop and both the abscissa length and the field-intensity monotonously decrease. These phenomena are demonstrated by a computer program, which is described in the layer C.

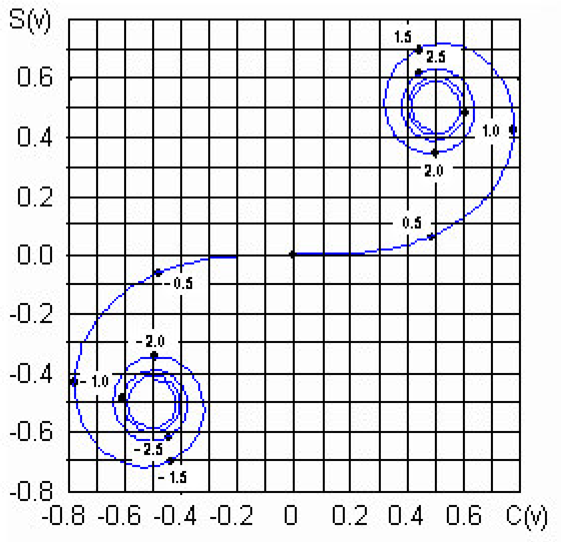

Fig. 2.1B.2 *Clothoid (number on curve are values of the argument v)*

In order to illustrate the influence of Fresnel zones to the field intensity behind the obstacle, we consider again the general result (2.1B.2). Instead of coordinates x, y of a facet dS, we introduce a radial distance $r_0 = \sqrt{x^2 + y^2}$ of the facet from the origin. Then,

$$E^{(P)} = C\frac{\exp[-jk(d_1+d_2)]}{d_1+d_2}\frac{j}{\lambda}\int_S \exp(-jar_0^2)dS. \qquad (2.1B.10)$$

We can see that contributions of facets in the different distance from the origin are of the different phase ($-ar_0^2$). In the distances r_{01}, r_{02}, ... r_{0n} given by

$$ar_{0n}^2 = n\pi \qquad (2.1B.11)$$

the phase is retarded for 180°. This is caused by enlarging the pointed line V - dS - P for 1/2. The radii r_{0n} in eqn. (2.1B.11) are the radii of the external border of given Fresnel zones. Substituting a from (2.1B.5) to (2.1B.11), we get a result, which is identical with (2.1A.4).

The fact that the phase of contributions of neighboring Fresnel zones (on the plane S) differs for 180° has got interesting causes. Let us imagine that the absorbing obstacle cover all the plane S. To this covered plane, a circular *window* with the center in O and with the arbitrary radius r_0 is cut. Though this *window*, wave propagates to the point P. Computing intensity $E^{(P)}$, we use eqn. (2.1B.10) and thanks to the rational symmetry of the window, we perform integration on S (over *window*) in polar coordinates: $dS = r_0\, dj dr_0$:

$$E^{(P)} = C\frac{\exp[-jk(d_1+d_2)]}{d_1 d_2}\frac{j}{\lambda}\int_0^{r_0}\int_0^{2\pi} r_0 \exp(-jar_0^2)\,\mathrm{d}\varphi\,\mathrm{d}r_0 . \qquad (2.1B.12)$$

The integral is easily solvable using the substitution $r_0^2 = \mathrm{r}$. Inscribing by

$$E_0 = C\frac{\exp[-jk(d_1+d_2)]}{d_1 d_2} \qquad (2.1B.13)$$

the free-space field intensity in P, following results are obtained for different *window* radii r_0:

Tab. 2.1B.1 *Intensity values for different window radios*

Window radio r_0	Zones	Intensity $E^{(P)}$
∞	All the space free	E_0
$0.58\, r_{01}$	Apart of the 1st Fresnel zone free	E_0
r_{01}	All the 1st Fresnel zone free	$2E_0$
r_{02}	The 1st Fresnel zone and the 2nd one free	0

Observing results, we might conclude that field intensity E_0 we measure in P without the obstacle present, is created by contributions of a free part of the first Fresnel zone and that contributions of the rest of S mutually eliminate. Unfortunately, the conclusion is not correct (we cannot determine the part of Fresnel zone producing the contributions which are not eliminated). On the other hand, covering single zones can suppress the mutual elimination and field intensity can be increased in P that way. Covering all the even Fresnel zones (all the odd ones) yields $E^{(P)} = \infty$. Such coverage acts as a burning glass, which focuses the wave to point P.