2.5 Wave propagation in layered media

Advanced theory

In the layer A, we introduced two approaches, which enable to solve electromagnetic wave propagation in a layered medium. The first approach was based on the distribution of field intensities and on the basic boundary condition. The second approach was based on the transformation of reflection coefficient and was identical with the method, which is used for solving similar tasks on transmission lines, in a fact. In this layer, we are going to introduce another approach, which makes the solution more formal, and therefore, it is suitable for more complicated situations.

The explanation is done for three media only (two boundaries). The situation is depicted in fig. **2.5B.1**. We can simply imagine multiple reflections appearing in the structure. Therefore, there are an infinite number of waves propagating in each layer in both directions. All the waves are coherent, and therefore, all the waves propagating in a given direction interfere into a single harmonic wave. Hence, only two waves in each medium are considered as depicted in fig. **2.5B.1**. The media are denoted as 1, 2, and 3. Next, A is the surface of the boundary 1-2 in the medium 1; B is the surface of the boundary 1-2 in the medium 2, etc. Using indexes A, B, C, D, we denote the surface, where the intensity is computed, and using indexes in brackets (p), (z), we denote the direction of propagation (forward wave, backward wave). E.g., $E_B^{(z)}$ denotes the intensity of the backward wave on the surface B, i.e. on the boundary 1-2 in the medium 2. The layered medium consists of boundaries and layers; we examine separately the wave transition through the boundary and through the layer.

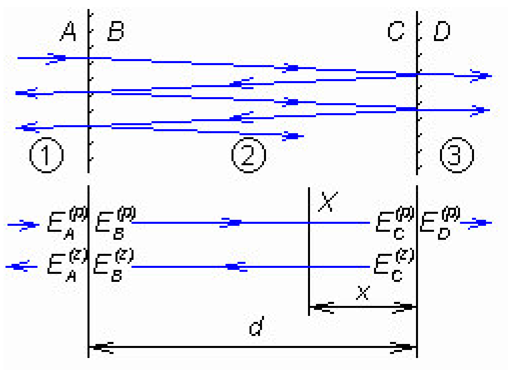

Fig. 2.5B.1 *Wave propagation in layered medium*

We consider the boundary 1-2 and express wave intensities, which travel outwards this boundary. The reflected wave of the intensity $E_A^{(z)}$ is created by the reflection of the incident wave $E_A^{(p)}$, and by the transition of the wave $E_B^{(z)}$ from the medium 2 to the medium 1. Similarly, the wave of the intensity $E_B^{(p)}$ is created by the transition of the wave $E_A^{(p)}$ from the medium 1 to the medium 2, and by the transition of the wave $E_B^{(z)}$. This is expressed by the equation:

$$E_A^{(z)} = \rho_{12} E_A^{(p)} + \tau_{21} E_B^{(z)}, \quad E_B^{(p)} = \tau_{12} E_A^{(p)} + \rho_{21} E_B^{(z)} \qquad (2.5B.1)$$

where ρ_{12} is reflection coefficient of the wave propagation in the medium 1 and reflecting from the boundary with the medium 2, and ρ_{21} is reflection coefficient of the wave propagating in the medium 2 and reflecting from the boundary with the medium 1. Similarly, coefficients of transmission are indexed as τ_{12} and τ_{21}. Eqn. (2.5B.1) can be rewritten to the matrix form:

$$\begin{bmatrix} E_A \\ E_B \end{bmatrix} = \begin{bmatrix} \rho_{12} & \tau_{21} \\ \tau_{12} & \rho_{21} \end{bmatrix} \begin{bmatrix} E_B \\ E_A \end{bmatrix}. \qquad (2.5B.2)$$

The right-hand side matrix, which is composed of coefficients of reflection and transmission, is called the scattering matrix of the boundary. The scattering matrix can be simply built, and moreover, their elements are of the obvious physical meaning (coefficients of reflection and transmission). Unfortunately, eqn. (2.5B.2) is not suitable for the solution of the problem because intensities of input and output waves are mixed in input and output column matrices. Eqn. (2.5B.2) is therefore solved for $E_A^{(p)}$ and $E_A^{(z)}$. Rearranging the relation, we consider ρ_{21} = -ρ_{12} and $\tau_{12}\,\tau_{21}$ - $r_{12}\,\rho_{21}$ = 1. The result is expressed in the matrix form:

$$\begin{bmatrix} E_A^{(p)} \\ E_A^{(z)} \end{bmatrix} = \frac{1}{\tau_{12}} \begin{bmatrix} 1 & \rho_{12} \\ \rho_{12} & 1 \end{bmatrix} \begin{bmatrix} E_B^{(p)} \\ E_B^{(z)} \end{bmatrix}. \qquad (2.5B.3)$$

The right-hand matrix is called the *cascade boundary matrix*. The left-hand column matrix consists of input-wave intensities, and the right-hand column matrix consists of output-wave intensities. The cascade matrix is suitable for cascade connection of elements (two-ports), which are present even in our task. If an arbitrary number of elements are connected into the cascade, then the cascade matrices are simply mutually multiplied.

Now, the cascade matrix of the dielectric layer has to be derived. This aim can be met without any need for computing the scattering matrix. In the dielectric layer, there are two traveling waves propagating in opposite directions. Obviously,
$E_B^{(z)} = E_C^{(z)} exp(-jk_2 d)$, $\quad E_B^{(p)} = E_C^{(p)} exp(+jk_2 d)$, and therefore

$$\begin{bmatrix} E_B^{(p)} \\ E_B^{(z)} \end{bmatrix} = \begin{bmatrix} \exp(+jk_2 d) & 0 \\ 0 & \exp(-jk_2 d) \end{bmatrix} \begin{bmatrix} E_C^{(p)} \\ E_C^{(z)} \end{bmatrix}, \qquad (2.5B.4)$$

where k_2 is wave number in the second medium.

In the system of plan-parallel layers, single boundaries and layers are lined in parallel way. The final equation can be therefore obtained by multiplying respective matrices. For the situation from fig. **2.5B.1**, we get:

$$\begin{bmatrix} E_A^{(p)} \\ E_A^{(z)} \end{bmatrix} = \frac{1}{\tau_{12}} \begin{bmatrix} 1 & \rho_{12} \\ \rho_{12} & 1 \end{bmatrix} \begin{bmatrix} \exp(+jk_2 d) & 0 \\ 0 & \exp(-jk_2 d) \end{bmatrix} \frac{1}{\tau_{23}} \begin{bmatrix} 1 & \rho_{23} \\ \rho_{23} & 1 \end{bmatrix} \begin{bmatrix} E_D^{(p)} \\ 0 \end{bmatrix}. \quad (2.5B.5)$$

The final result supposes a traveling wave in the third medium. In the opposite case, the intensity $E_D^{(z)}$ appears in the last column matrix. The way of extending the principle to an arbitrary number of layers is obvious.

Rewriting (2.5B.5), we get:

$$E_A^{(p)} = \frac{e^{jk_2 d} + \rho_{12}\rho_{23} e^{-jk_2 d}}{\tau_{12}\tau_{23}} E_D^{(p)}, \quad E_A^{(z)} = \frac{\rho_{12} e^{jk_2 d} + \rho_{23} e^{-jk_2 d}}{\tau_{12}\tau_{23}} E_D^{(p)}. \quad (2.5B.6)$$

The ratio $E_A^{(z)}/\ E_A^{(p)}$ gives the real value of the reflection coefficient on the boundary 1-2. The ratio $E_D^{(p)}/\ E_A^{(p)}$ equals to the total transmission coefficient. Both the quantities can be computed using the last equations.

Let us note that in the case of the normal incidence of a perpendicularly polarized wave to a *single* boundary (no reflected wave can exists behind that), following relations are valid for the reflection coefficient and the transmission one:

$$\rho_{\perp} = \frac{Z_{02} - Z_{01}}{Z_{02} + Z_{01}}, \quad \tau_{\perp} = 1 + \rho_{\perp} = \frac{2Z_{02}}{Z_{02} + Z_{01}}. \quad (2.5B.7)$$

The described method is suitable for more complicated structures. Common tasks can be numerically solved using Smith chart or directly considering boundary conditions on the boundary (see layer A.)

As already mentioned in the introduction, provision of the reflection-free transition of the electromagnetic wave from one medium to another one is an example of the application of layered media. Such requirement can appear when exciting waves in the ground or in the other media, when exciting waves in human body, at dielectric radomes of antennas, etc.; the wave is required to enter the medium and to exit it with the same intensity and the same equiphase surface. Covering the boundary between the media by one or several dielectric layers of exactly given permittivity and thickness can fulfill the requirement. The layers play the role of the matching circuitry (transformer) in analogy to the transmission line. In both the cases, electromagnetic wave propagates in the structure. On the transmission line, voltages and currents represent the wave. In the dielectric layer, the wave is represented by electric-field intensity and magnetic-field intensity. Voltage U corresponds to electric intensity E, and current I corresponds to magnetic intensity H. Characteristic impedance of the transmission line corresponds to wave impedance of the medium (layer). The difference between the above-described cases is hidden in the structure of circuits. On the transmission line, matching circuitry could consist not only of cascade-connected segments of the transmission line but too of parallel or serial shunts. Matching layers can be connected in cascade only. The reason for that is fundamental. In the part of the transmission line, where the parallel reactance is connected, a current jump appears (a part of current flows to the reactance). In the layered equivalent, a jump of magnetic-field intensity should appear in the respective part of the boundary. This is impossible from the physical point of view (it contradicts the basic boundary condition). In general, jump of magnetic-field intensity H can appear on the boundary if electric current is admitted. Then, the boundary condition is of the form $\mathbf{n} \times \mathbf{H}_1 = \mathbf{K} + \mathbf{n} \times \mathbf{H}_2$. Indexes distinguish between the first medium and the second one, vector products of the normal $\mathbf{n}$ result in tangential components and $\mathbf{K}$ [A m^{-1}] is surface density of electric current on the boundary. In practical life, this solution can be used. However, the elected technology has to admit the flow of electric currents on the surface of the boundary.

Routinely, we can design matching circuits consisting of a single one-quarter-wavelength thick dielectric layer or of a first dielectric layer of a given thickness plus of a second one, which is one-quarter-wavelength thick. No matter whether segments of the transmission line or dielectric layers are used. If efficient, Smith chart can be used.

The above-described matching circuits can be modeled by the computer program, which is described in the layer C. The program cannot be used for the design of those circuits, it analyses a circuitry of given parameters (permittivity, thickness of layers) and computes the distribution of E and H in layers, reflection coefficient and standing wave ratio in various points of the structure. Observing numerical values, the circuit operation can be investigated. The matching circuits can consists of up to four dielectric layers. The operation of the circuitry can be observed in a wider frequency band.

Frequency filters are another application area of layered media. As a basic element, the half-wavelength-thick dielectric layer is used. This layer (in analogy with the half-wavelength-long transmission line) transforms intensities and impedances in the ratio 1:1. Other words, the wave reflected at the beginning of the layer cancels out with the wave reflected from the end of layer. If the incident wave is of different frequency (the layer is not half-wavelength thick), the total cancellation does not appear. The layer reflects the part of energy, and therefore, the transmitted energy is lower. In front of the layer, the standing wave appears.

In the practical life, filters are composed of several half-wavelength-thick layers, which are mutually separated by layers of another permittivity and thickness (*l*/4, e.g.).

Properties of layered structures as filters can be demonstrated by another program in the layer C. In this layer, even the more detailed description of the program is given.