4.2 Mutual impedance

Basic theory

Radiating elements (dipoles, loops, etc.) are usually grouped into an antenna array and fed from a common source in order to primarily reach the desired directivity pattern of the radiation.

Designing the feeding system of an antenna array, even the mutual influence among the antenna elements has to be considered and a true value of the impedance at the input of each antenna element has to be determined. Knowledge of these entries enables to determine input current of each antenna element and even currents on the feeding ports of the array. Moreover, the structure of the feeding system can be modified to reach a suitable value of the input impedance of the antenna array in its whole.

In this chapter, we concentrate on the method of calculating input impedance of a linear antenna (dipole) by the method of induced electromotoric forces. The approach is then going to be generalized in order to compute impedances of single elements in the antenna array.

Radiation impedance Z_Σ of the antenna can be considered as a proportionality constant between the complex power P_Σ, which is radiated by the antenna, and the squared value of the related current I

$$P_\Sigma = Z_\Sigma |I|^2 = Z_{\Sigma vst}|I_{vst}|^2 = Z_{\Sigma m}|I_m|^2. \qquad (4.2A.1)$$

As a related current, we consider the input current of the element I_{vst} or the value of the current I_m in the maximum of the standing wave. Since currents I_{vst} and I_m of the same antenna (under identical conditions) usually differ, but the radiated power stays the same, even the values of the radiation impedance differ - the value of $Z_{\Sigma vst}$ related to the input current I_{vst} and the value of $Z_{\Sigma m}$ related to the current in its maximum I_m are not the same.

Neglecting losses of the antenna, the radiation impedance $Z_{\Sigma vst}$ equals to the input impedance of the antenna Z_{vst} computed as a ratio of input voltage U_{vst} and input current I_{vst}

$$U_{vst} = Z_{vst} I_{vst}. \qquad (4.2A.2)$$

The above-defined values of input impedance Z_{vst} are usually provided for basic types of antennas. These values are valid for antennas placed in *free space*, i.e. in satisfactorily long distance from other antennas or objects. A detailed algorithm of computing Z_{vst}, which is based on the radiated power, is given in the layer B.

In an antenna array, single antenna elements mutually influence themselves and impedance of each element depends on the type of elements in the surrounding, on the way of their positioning and feeding. In order to evaluate and compute the mutual influence, we change our view to the equation (4.2A.2).

The current I is assumed to be excited in the antenna any way. The current I is characterized by the current I_{vst} and by a function of current distribution. Antenna radiates and creates a given intensity of electric field E in its surrounding, which is proportional to the current on the antenna. A certain field intensity E_t appear even on the surface of the radiating antenna. At the same moment, antenna acts as the receiving antenna, and the receive results in a given voltage at the input port of the antenna. Voltage U_{vst} in eqn. (4.2A.2) can be therefore considered as a voltage, produced on the antenna by the receive of the own radiation. This own radiation is proportional to the current magnitude I_{vst} at its input and the quantity Z_{vst} (self-impedance) plays the role of the proportionality constant.

The above-described consideration can be simply applied to the whole antenna array. Even in the array, there is a certain field intensity E_t on the surface of each antenna element. This field is not created not only by the self-radiation of the antenna element but too by the radiation of the other elements.

In fig. **4.2A.1**, the antenna array consisting of n elements (dipoles), which are fed by currents $I_{vst\,i}$ on their input terminals, is depicted. In analogy to (4.2A.2), input voltages of antenna elements are given by the following set of equations

$$\begin{aligned} U_{vst1} &= Z_{11} I_{vst1} + Z_{12} I_{vst2} + \dots\dots + Z_{1n} I_{vstn} \\ U_{vst2} &= Z_{21} I_{vst1} + Z_{22} I_{vst2} + \dots\dots + Z_{2n} I_{vstn} \\ &\vdots \\ U_{vstn} &= Z_{n1} I_{vst1} + Z_{n2} I_{vst2} + \dots\dots + Z_{nn} I_{vstn} \end{aligned} \qquad (4.2A.3)$$

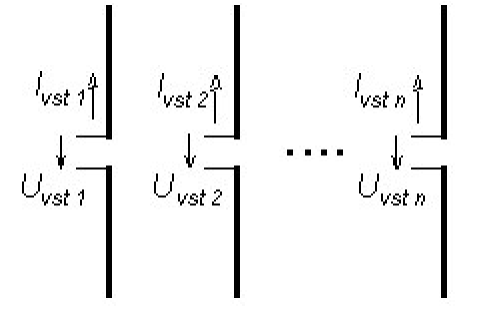


Fig. 4.2A.1 *Array of n dipoles*

Impedance coefficients Z_{jk} (4.2A.3) express the mutual coupling between j-th and k-th element and are called mutual impedance. Mutual impedance is a complex quantity fulfilling $Z_{jk} = Z_{kj}$. Its magnitude depends on the shape, on the dimensions and on the mutual position of antenna elements, and even on their current distribution.

Coefficient Z_{jj} is called self-impedance and determines relation between the current and the voltage at the input of antenna elements out of the

Copyright © 2010 FEEC VUT Brno All rights reserved.

array and equals to the radiation impedance of a separated j-th antenna element in free space.

Typical dependency of the components of the mutual impedance $Z_{jk} = R_{jk} + jX_{jk}$ on the distance d (multiplied by wave-number $k = 2\pi/\lambda$) between two parallel dipoles of the same length is depicted in fig. **4.2A.2**.

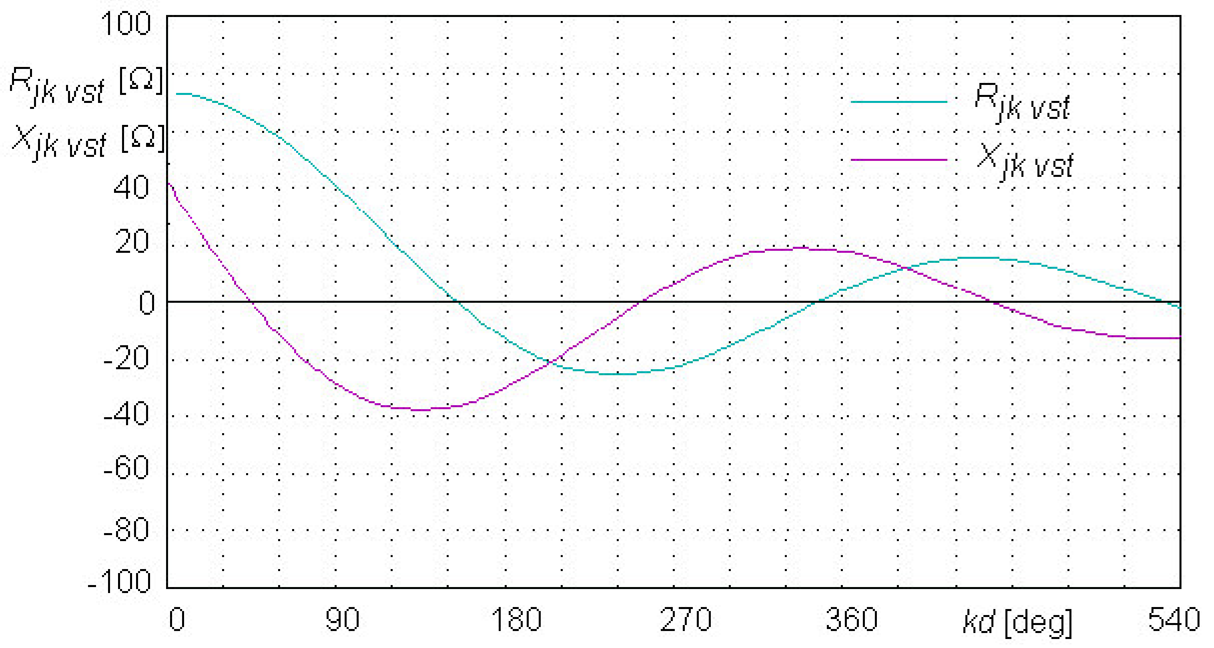


Fig. 4.2A.2 *Mutual impedance between two identical parallel dipoles in the distance d. Length of one half of the dipole is $l = \lambda/4$.*

Magnitude of mutual impedance depends on dimensions and on the distance of antenna elements. Real and imaginary component of mutual impedance can be both positive and negative and its maximum values decrease when the distance between elements d rises (the influence of very distant elements negligible).

Values of mutual impedance, which are provided for technical needs in charts, are usually related to the current in maximum. Re-computation to input terminals can be done considering the sine current distribution on the antenna

$$Z_{jkvst} = \frac{Z_{jkm}}{\sin^2(kl)}, \qquad (4.2A.4)$$

where $kl = (2\pi/\lambda)\, l$, if antenna length l is not close to the integral multiple of $\lambda/2$.

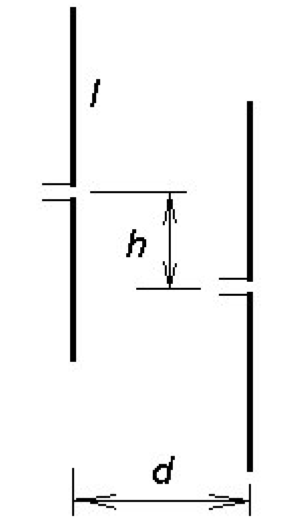


Fig. 4.2A.3 *Couple of parallel dipoles*

More accurate values of mutual impedance can be computed using the program from the layer C. The program computes values of mutual impedance Z_{jk}, related to the maximum current or to the input one for a couple of two parallel dipoles of the same length and of the same standing wave on the wire of the dipoles. The length of the dipole l, the distance between dipoles d and the magnitude of the axial shift of the feeding ports of the dipoles h (fig. **4.2A.3**) have to be entered when multiplied by wave-number $k = 2\pi/\lambda$. Graphic representation of the magnitude of both components of mutual impedance R_{jk} and X_{jk} on the variation of a elected quantity (l, d or h) is suitable when the influence of the spatial arrangements of the antenna array to the impedance relations is investigated.

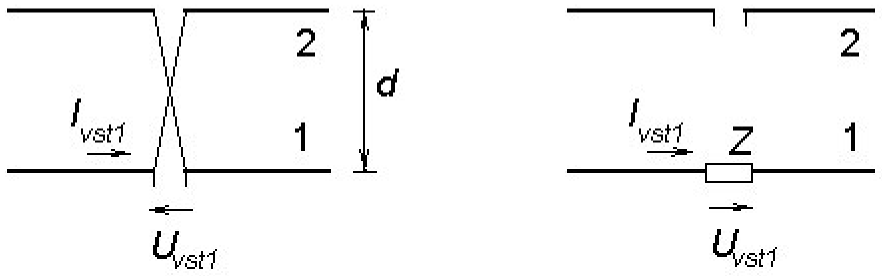
The same way, components of the radiation impedance Z_{ii} at the input of an isolated antenna element can be computed if *the distance between elements d* is put to be equal to the radius of the antenna wire a.

Computing voltages and currents in the system of n antenna elements, $2n$ independent quantities (n voltages and n currents) have to be determined. Considering the way of feeding, other (n-1) equations can be built. For elected magnitude of a single voltage (current), the ($2n$-1) unknown quantities can be computed. In antenna arrays, where single antenna elements are fed by the system of transmission lines, computations are complicated by the fact that impedance of each element depends on currents in other elements. That way, relations on transmission lines and radiators are mutually coupled. In case of single fed element, computations are rather simple.

Deriving equations (4.2A.3) for a given situation, we have to respect the fact that orientation of voltage $U_{vst\,i}$ and of a current $I_{vst\,i}$ correspond to the feeding of an element by a generator. In case of a passive element (voltage on the input port is caused by the current I_{vst} flowing through the load Z of the input port) is the orientation of the voltage opposite, and therefore, the sign has to be changed in (4.2A.3). For two typical

Copyright © 2010 FEEC VUT Brno All rights reserved.

situations, corresponding sets of equations are given in fig. **4.2A.4**.



$$U_{vst1} = Z_{11} I_{vst1} + Z_{12} I_{vst2}$$
$$U_{vst2} = Z_{21} I_{vst1} + Z_{22} I_{vst2}$$
$$U_{vst1} = U_{vst2}$$

a)

$$U_{vst1} = Z_{11} I_{vst1} + Z_{12} I_{vst2}$$
$$U_{vst2} = Z_{21} I_{vst1} + Z_{22} I_{vst2}$$
$$U_{vst1} = -Z I_{vst1}$$

b)

Obr. 4.2A.4 *Voltages and currents in antenna systems. **a)** two dipoles fed by a transmission line (d=λ/2), **b)** system consisting passive element*

Set (4.2A.3) enables to compute the ratio between any couple of unknown quantities. Input impedance of i-th element $Z_{vst\,i}$ is obtained when dividing i-th eqn. (4.2A.3) by input current of i-th element $I_{vst\,i}$

$$Z_{vsti} = \frac{U_{vsti}}{I_{vsti}} = Z_{i1}\frac{I_{vst1}}{I_{vsti}} + Z_{i2}\frac{I_{vst2}}{I_{vsti}} + \ldots\ldots + Z_{ii} + \ldots\ldots + Z_{ni}\frac{I_{vstn}}{I_{vsti}}. \qquad (4.2A.5)$$

Impedance of every element of an antenna array is therefore given by the sum of its own impedance Z_{ii} and of submissions of the other elements, depending on the product of mutual impedances Z_{jk} and respective currents (amplitudes and phases) flowing through the input ports of those elements. The change of feeding any antenna element therefore causes the change of impedances of all the antenna elements in the array. As an example, let us consider results obtained for the arrays depicted in fig. **4.2A.4**. In the case of the couple of dipoles fed by the crossed transmission line of the length $d = \lambda/2$ (fig. **4.2A.4a**), currents of the same magnitude and the same phase flow in both the dipoles, and both of the dipoles are of the input impedance

$$Z_{vst1} = Z_{vst2} = Z_{11} + Z_{12}\frac{I_{vst2}}{I_{vst1}} = Z_{11} + Z_{12}. \qquad (4.2A.6)$$

Impedance on the port of the dipole "1" equals to the half of the value Z_{vst1}. For the couple of the half-wavelength-long dipoles (kl = 90°), we ge Z_{11} = (73,1 + j42,5) Ω and Z_{12} = (-12,5 + j30) Ω. Assuming in-phase feeding (fig. **4.2A.4a**), the resultant value of the impedance of both the dipoles equals to Z_1 = (60,6 + j12,5) Ω. Considering anti-phase feeding, the resultant input impedance equals to the difference of Z_{11} and Z_{12}, which yields the result Z_1 = (85,6 + j72,5) Ω. The same result is obtained for the dipole in the distance λ/4 from the planar reflector. Impedance on the port "1" in fig. **4.2A.4a** equals to the half of the computed value Z_1, i.e. Z_{soust} = (30,3 + j6,2) Ω.

Input impedance of the dipole "2" in fig. **4.2A.4b** is given by the relation

$$Z_{vst2} = Z_{22} - \frac{Z_{21}^2}{Z + Z_{11}}. \qquad (4.2A.7)$$

Substituted values of both the impedances Z_{11} and Z_{12} have to be related to the input current of the dipole I_{vst} in both the cases.

Copyright © 2010 FEEC VUT Brno All rights reserved.