7.1 Gaussian beam

Basic theory

Investigating parameter variation of coherent optic beam, which passes an optical system, the simplest laser beam TEM_{00} is assumed. When leaving the source, the beam is *parallel* to the optical axis (its planar equiphase surface is perpendicular to the direction of propagation). Even if the beam is not modulated, transversal distribution of field intensity is not constant.

Waves, which equiphase surface normal declines for a very small angle form the optical axis (axis z), are called paraxial waves. Those waves have to meet the equation

$$\nabla_T^2 A - j2k\frac{\partial A}{\partial z} = 0, \tag{7.1A.1}$$

where

$$\nabla_T^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

is the transversal part of Laplace operator, z is a coordinate of the longitudinal axis, k denotes wave number, A is field intensity.

Gaussian beam is one of possible solutions of eqn. (7.1A.1). Power of Gaussian beam is concentrated into a narrow cone. Field distribution in an arbitrary transversal plane is given by a circularly symmetric Gaussian function; the axis of symmetry is identical with the optical axis. The width of Gaussian beam is minimal is a so-called neck (here, the equiphase surface is planar). The width gradually increases on both the sides of the neck (equiphase surfaces are gradually curved).

As already noted, transversal distribution of field intensity of the basic mode TEM_{00} is described by Gaussian function (the maximum intensity on the axis gradually decreases towards margin):

$$E(x, y) = E_{max}\exp\left(-\frac{\rho^2}{a_0^2}\right) = E_{max}\exp\left(-\frac{x^2+y^2}{a_0^2}\right), \tag{7.1A.2}$$

where r denotes the radial distance of the point (x, y) from the beam axis, and a_0 is so-called beam radius (a radial distance from the beam axis, where field intensity decreases to the value E_{max}/e; $e = 2.718...$.

Complex amplitude

Assume a paraxial plane wave exp(-jkz), where $k = 2\pi/\lambda$ is wave number, λ denotes wavelength and z is optic axis coordinate. The wave is modulated by the envelope $A(r, z)$, which varies relatively slowly in the direction of the optical axis z. Then, the complex amplitude meets

$$U(r) = A(r, z)\exp(-jkz). \tag{7.1A.3}$$

The envelope is assumed to stay approximately constant when the distance changes for $\Delta z = \lambda$. We speak about a local plane wave, which equiphase surface normals form paraxial beams. Rearranging equations, the definition of the complex envelope of Gaussian beam is obtained:

$$A(r, z) = \frac{A_1}{q(z)}\exp\left[-jk\frac{\rho(r)^2}{2q(z)}\right], \quad q(z) = z + jz_0, \quad \rho^2 = x^2 + y^2. \tag{7.1A.4}$$

Here, z_0 is Rayleigh distance.

In order to separate the amplitude and the phase of a complex envelope, function $1/q(z)$ is rewritten to the real part $R(z)$ and to the imaginary one, which is represented by the function $W(z)$. Hence,

$$\frac{1}{q(z)} = \frac{1}{R(z)} - j\frac{1}{\pi W^2(z)}. \tag{7.1A.5}$$

The function $R(z)$ describes the half-width of Gaussian beam, and $W(z)$ is curvature radius of the beam equiphase surface.

For beam parameters, we can further define:

$$W(z) = W_0\sqrt{1 + \left(\frac{z}{z_0}\right)^2}, \tag{7.1A.6}$$

$$R(z) = z\left[1 + \left(\frac{z}{z_0}\right)^2\right], \tag{7.1A.7}$$

$$W_0 = \sqrt{\frac{\lambda z_0}{\pi}}. \tag{7.1A.8}$$

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Properties of Gaussian beam

Gaussian beam is uniquely determined by the following parameters:

- Intensity of radiation
- Power of the beam
- Radius of the beam

The above-given parameters are going to be explained more in detail now.

Intensity of radiation

Intensity of radiation is a function of the axial distance z and the radial one $\rho = (x^2 + y^2)^{1/2}$,

$$I(\rho, z) = I_0 \left[\frac{W_0}{W(z)}\right]^2 \exp\left[-\frac{2\rho^2}{W^2(z)}\right]. \quad (7.1A.9)$$

At the beam axis ($\rho = 0$), intensity is of the maximal value I_0 for $z = 0$. Increasing z, intensity gradually decreases, and for $z = \pm z_0$, the intensity is of the half of the maximum value I_0.

Power of the beam

The total power, which is transmitted by the beam, is given by the integral of the product of the beam intensity and its transversal surface

$$P = \int_0^{\infty} I(\rho, z) 2\pi\rho d\rho. \quad (7.1A.10)$$

At the axis, we get

$$P = \frac{1}{2} I(0,z)\pi[W(z)]^2. \quad (7.1A.11)$$

Inside a circle of the radius $\rho_0 = W(z)$, approximately 86 % of the total power is transmitted. Through a circle of the radius 1.5 $W(z)$, approximately 99 % of power passes

Since Gaussian beams are often characterized by the transmitted power P, the intensity I is useful to be expressed as a function of P

$$I(\rho, z) = \frac{2P}{\pi W^2(z)} \exp\left[-\frac{2\rho^2}{W^2(z)}\right]. \quad (7.1A.12)$$

Radius of the beam

In every transversal plane, the intensity is maximal at the optical axis (z). Since most power propagates in the area of the radius $W(z)$, we understand $W(z)$ as the radius of the beam. Dependency of the radius on the coordinate z is given by the relation

$$W(z) = W_0 \sqrt{1 + \left(\frac{z}{z_0}\right)^2}. \quad (7.1A.13)$$

In the plane z = 0, the radius is of the minimal walue W_0, which is called the position of the maximum necking of the beam.

Further information can be found in **[16]**.

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