7.2 Passage of Gaussian beam through optic elements

Basic theory

In this paragraph, influence of various optical elements to the parameters of passing Gaussian beam is studied. If Gaussian beam propagates through a circularly symmetric system, which consists of a sequence of optical elements, then the nature of a beam stays Gaussian, and only its parameter change. This change of parameters can be simply computed by matrix optics.

Parameters of Gaussian beam can be observed depending on the position of the observation point (a coordinate on the optical axis) and on the angle between the position vector of the observation point and the optical axis. In paraxial approximation, the position and the angle are mutually associated by two algebraic equations. The optical system is therefore described by the matrix of size 2 × 2. This matrix is called the matrix of beam. In general:

$$\begin{aligned} y_2 &= Ay_1 + B\Theta_1 \\ \Theta_2 &= Cy_1 + D\Theta_1 \end{aligned} \qquad \begin{bmatrix} y_2 \\ \Theta_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} y_1 \\ \Theta_1 \end{bmatrix}. \qquad (7.2A.1)$$

Here, y_1 is position of the input of an optical element, y_2 is position of the output, θ_1 is an angle at the input of the optical element, and θ_2 is an angle at the output (with respect to the optical axis).

ABCD law

Let us denote parameters of Gaussian beam at the input plane of the optical element as q_1, and parameters of Gaussian beam at the output plane of the optical element as q_2. The optical element is described by the matrix [A, B; C, D]. We can show that all the introduced quantities are related by the equation

$$q_2 = \frac{Aq_1 + B}{Cq_1 + D}. \qquad (7.2A.2)$$

Since parameters q determine the half-width of the Gaussian beam W and its curvature radius R, eqn. (7.2A.2), which is called ABCD law, describes the transform of the Gaussian beam by an arbitrary paraxial optical system.

Transmission matrices of simple optical elements

In this paragraph, matrices of the most frequently used optical elements are given.

Propagation in vacuum

A distance d in free space can be considered as the simplest optical element. Since wave propagates along beams, coordinates of the beam, which passes through the distance d, change according to the equation $y_2 = y_1 + \theta_1 d$ and $\theta_2 = \theta_1$. The transmission matrix **M** is therefore

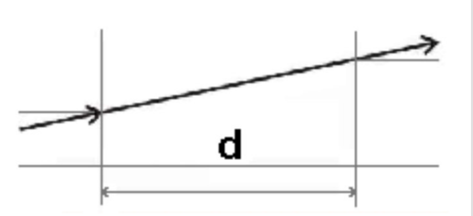


Fig. 7.2A.1 *Vacuum of the length d*

$$M = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}. \qquad (7.2A.3)$$

Refraction on planar boundary

On planar boundary of two media of refraction indexes n_1 and n_2, angles of the beam change according to the Snell law

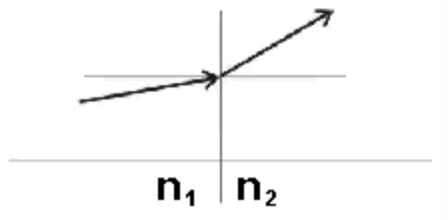


Fig. 7.2A.2 *Refraction on planar boundary of two media of different refraction index*

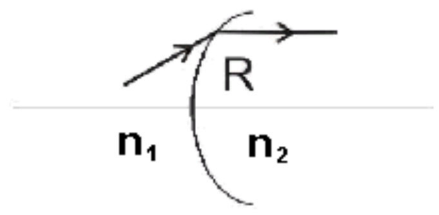
$$n_1 \sin(\Theta_1) = n_2 \sin(\Theta_2). \quad (7.2A.4)$$

In paraxial approximation, we have $n_1\theta_1 \approx n_2\theta_2$, and therefore, position of the beam stays unchanged, i.e. $y_2 = y_1$. The transmission matrix **M** is therefore

$$M = \begin{bmatrix} 1 & 0 \\ 0 & n_1/n_2 \end{bmatrix}. \quad (7.2A.5)$$

Refraction on spherical boundary

Relation between angles θ_1 and θ_2 for paraxial beams refracting on a spherical boundary of two media is given by



convex boundary: R>0
concave boundary: R<0

Fig. 7.2A.3 *Refraction on spherical boundary of two media of different refraction index*

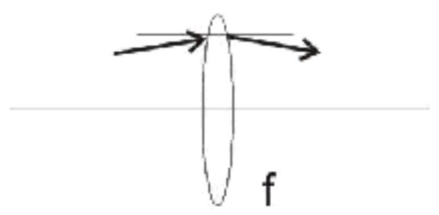
$$\Theta_2 \approx \frac{n_1}{n_2}\Theta_1 - \frac{n_2 - n_1}{n_2 R} y. \quad (7.2A.6)$$

The distance of the beam from the axis stays unchanged, i.e. $y_2 \approx y_1$. The transmission matrix **M** is therefore:

$$M = \begin{bmatrix} 1 & 0 \\ -\frac{(n_2 - n_1)}{n_2 R} & \frac{n_1}{n_2} \end{bmatrix}. \quad (7.2A.7)$$

Passage through a thin lens

Relation between angles θ_1 and θ_2 for paraxial beams, which pass through a thin lens of the focus distance f, is:



convex lens: f>0
concave lens: f<0

Fig. 7.2A.4 *Passage through a lens of focus distance f*

$$\Theta_2 = \Theta_1 - \frac{y}{f}. \quad (7.2A.8)$$

The distance form the axis stays unchanged. The transmission matrix **M** is therefore:

$$M = \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix}. \quad (7.2A.9)$$

Reflection from planar mirror

Reflecting beam from a planar mirror, the position of a beam stays unchanged ($y_2 = y_1$). For angles, we get $\theta_2 = \theta_1$. Hence, the transmission matrix is unitary

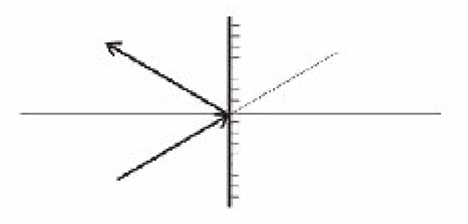
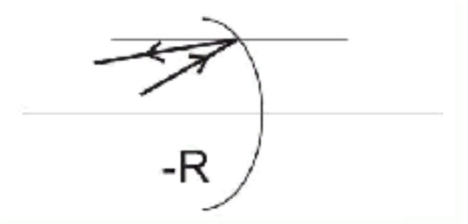


Fig. 7.2A.5 *Reflection from planar mirror*

$$M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \quad (7.2A.10)$$

Reflection from spherical mirror

Exploiting (7.2A.9), we obtain:



convex mirror:
concave mirror:

Fig. 7.2A.6 *Reflection from spherical mirror*

$$(-\Theta_2) + \Theta_1 \approx \frac{2y}{-R}, \quad (7.2A.11)$$

$$M = \begin{bmatrix} 1 & 0 \\ 2/R & 1 \end{bmatrix}. \quad (7.2A.12)$$

Transmission matrix of the sequence of optical elements

Sequence of optical elements, which are described by matrices $\mathbf{M}_1$, $\mathbf{M}_2$, ..., $\mathbf{M}_N$, is equivalent to a single optical element of the transmission matrix:

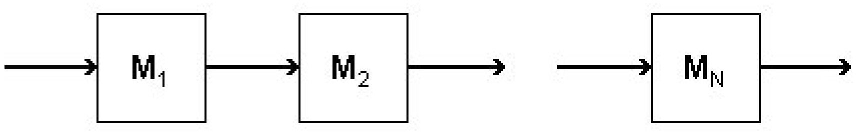


Fig. 7.2A.7 *Sequencing optical elements*

$$\mathbf{M} = \mathbf{M}_1 \cdot \mathbf{M}_2 \cdot \ldots \cdot \mathbf{M}_N \quad (7.2A.13)$$

Note the order of the matrix multiplication. The matrix of the element, which is entered by the beam first, appears on the right, and therefore, it multiplies the column vector of the incident beam first.

More information can be found in **[16]**.